Paying for Inattention

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Introduction

- Individuals often choose from a discrete set of actions
- Outcomes of these actions aren't always known, but can be acquired with some cost
- How much information is optimal to acquire?
- What is the best action given the information?

Model

→ We start with the model of Matejka and McKay (AER, 2014)

- The agent chooses an action from the set $A = \{1, 2, ..., N\}$
- The state of nature is a vector $\boldsymbol{v} \in \mathbb{R}^N$, prior $G \in \Delta(\mathbb{R}^N)$
- v_i is the payoff of action $i \in A$
- Let $V(B) := \max_{i \in A} \mathbb{E}_B[v_i]$

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Stage 1 Choose an information structure to maximize

$$\max_{F \in \Delta(\mathbb{R}^{2N})} \int_{\boldsymbol{v}} \int_{\boldsymbol{s}} V(F(\cdot|\boldsymbol{s}))F(d\boldsymbol{s}|\boldsymbol{v})G(d\boldsymbol{v}) - c(F)$$

$$\text{s.t.} \quad \int_{\boldsymbol{s}} F(d\boldsymbol{s}, \boldsymbol{v}) = G(\boldsymbol{v}), \forall \boldsymbol{v} \in \mathbb{R}^N$$

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Stage 2 Choose an action $a : \Delta(\mathbb{R}^N) \to A$ to maximize expected payoff given $F(\cdot|\boldsymbol{s})$

$$a(F) = \arg\max_{i \in A} \mathbb{E}_F[v_i]$$

Cost Function

- The entropy-based cost function

$$c(F) := \lambda \left(H(G) - \mathbb{E}_{\boldsymbol{s}} \left[H(F(\cdot|\boldsymbol{s})) \right] \right)$$

where

$$H(B) = -\sum_{k} P_k \log(P_k)$$

P_k is the probability of state k.

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- Cost in new terms

$$c(\mathcal{P}, G) = \lambda \left(-\sum_{i=1}^{N} \mathcal{P}_i^0 \log(\mathcal{P}_i^0) + \sum_{i=1}^{N} \int_{\boldsymbol{v}} \mathcal{P}_i(\boldsymbol{v}) \log \mathcal{P}_i(\boldsymbol{v}) G(d\boldsymbol{v}) \right)$$

Induced Problem

- Equivalent to initial problem

$$\max_{\mathcal{P}=\{\mathcal{P}_i(\boldsymbol{v})\}_{i=1}^N} \sum_{i=1}^N \int_{\boldsymbol{v}} v_i \mathcal{P}_i(\boldsymbol{v}) G(d\boldsymbol{v}) - c(\mathcal{P}, G)$$

subject to

$$\forall i: \qquad \mathcal{P}_i(\boldsymbol{v}) \geq 0, \qquad \forall \boldsymbol{v} \in \mathbb{R}^N \tag{1}$$

$$\sum_{i=1}^N \mathcal{P}_i(\boldsymbol{v}) = 1, \qquad \forall \boldsymbol{v} \in \mathbb{R}^N \tag{2}$$

Solve the Induced Problem

- Lagrangian

$$\mathcal{L}(\mathcal{P}) = \sum_{i=1}^{N} \int_{\boldsymbol{v}} v_i \mathcal{P}_i(\boldsymbol{v}) G(d\boldsymbol{v}) - c(\mathcal{P}, G) + \int_{\boldsymbol{v}} \xi_i(\boldsymbol{v}) \mathcal{P}_i(\boldsymbol{v}) G(d\boldsymbol{v}) - \int_{\boldsymbol{v}} \gamma_i(\boldsymbol{v}) \left(\sum_{i=1}^{N} \mathcal{P}_i(\boldsymbol{v}) - 1 \right) G(d\boldsymbol{v})$$

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- FOC

$$\mathcal{P}_i(\boldsymbol{v}) : \quad v_i + \xi_i(\boldsymbol{v}) - \gamma(\boldsymbol{v}) + \lambda(\log \mathcal{P}_i^0 + 1 - \log \mathcal{P}(\boldsymbol{v}) - 1) = 0$$

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$$\mathcal{P}_i(\boldsymbol{v}): \quad v_i + \xi_i(\boldsymbol{v}) - \gamma(\boldsymbol{v}) + \lambda(\log \mathcal{P}_i^0 + 1 - \log \mathcal{P}(\boldsymbol{v}) - 1) = 0$$

- Solving for $\mathcal{P}_i(\boldsymbol{v})$ and using $\sum_{i=1}^{N} \mathcal{P}_i(\boldsymbol{v}) = 1$ we get

$$\mathcal{P}_i(\boldsymbol{v}) = \frac{\mathcal{P}_i^0 e^{v_i/\lambda}}{\sum_{j=1}^{N} \mathcal{P}_i^0 e^{v_j/\lambda}}$$

Result

- Probability of choosing an action $i \in A$ given the state $\boldsymbol{v}$

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- Some properties:
  - Adding an action k to the choice set can increase the likelihood of selecting action i - no RUM can provide this
  - Invariant to duplicate actions
  - Monotonicity in FOSD sense

EXAMPLE

Solve the Induced Problem

- Agent must choose between taking a red bus, a blue bus, or a train.

	State 1	State 2	State 3	State 4
red bus	0	1	0	1
blue bus	0	0	1	1
train	R	R	R	R
$G(\boldsymbol{v})$	$\frac{1}{4}(1+\rho)$	$\frac{1}{4}(1-\rho)$	$\frac{1}{4}(1-\rho)$	$\frac{1}{4}(1+\rho)$

Case 1: $R \geq 1$

Case 2: $R = 1/2$ and $\rho = -1$

Case 3: $\rho = 1$

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$G(\boldsymbol{v})$	0	1/2	1/2	0

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$G(\boldsymbol{v})$	1/2	0	0	1/2

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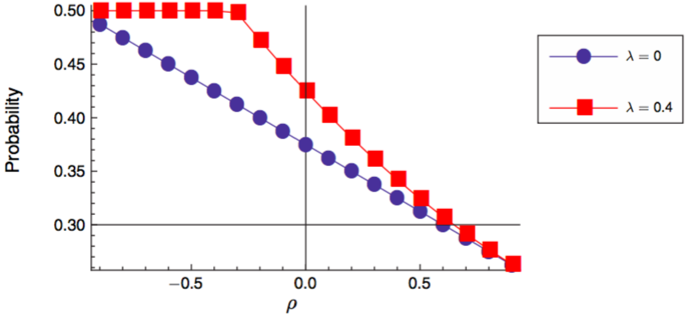
Probability of Choosing a Bus



Motivation

- There is no empirical evidence to support entropy based cost as a good approximation to the true information processing cost
- Study the tradeoff between attention and incentives
- Identify the attention cost function (shape but not the level)
- Attention and Risk aversion

Introduce Redistribution

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	b	w
Action B	Y	0
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- With some cost $q(x)$ agents can transform the state space

	b	w
Action B	Y	xY
Action W	xY	Y

The Problem

- Agent's maximization problem

$$\max_{x,p} \quad pY + (1-p)xY - q(x) - c(p)$$
$$\text{s.t.}$$
$$0 \leq x \leq 1$$
$$0 \leq p \leq 1$$

Results

- Targeted probability can be expressed in terms of parameters and optimal x^*

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- Derivative of the cost function

$$c'(p^*) = Y(1 - x^*)$$

Exogenous and Endogenous Transfers

Exogenous transfers:

- Choose some q function and fix some levels of x
- Ask subjects to perform the task under different levels of x

Endogenous transfers:

- Choose some q functions
- Ask subjects to do choose x for every q and execute the task with chosen level of x

Exogenous Transfers

- Task - count black and white balls
- Three levels of difficulty: low(65), medium(130), high(190)
- Four transfer levels: 0%, 35%, 65% and 100%
- Payoff pairs: $(\$20, \$0), (\$18, \$5), (\$14, \$7), (\$8, \$8)$

Sample Screen for Exogenous Transfers

Period: 1

If you SUCCEED in the task you will earn:

13.00

If you FAIL the task you will earn:

4.00

Reveal The Screen With Balls

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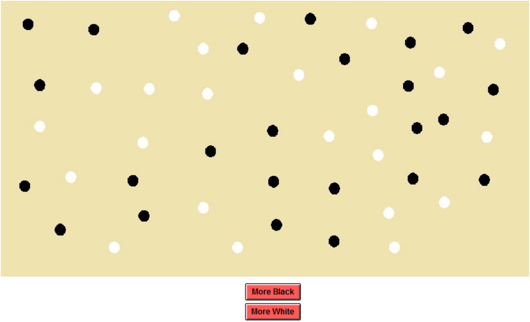
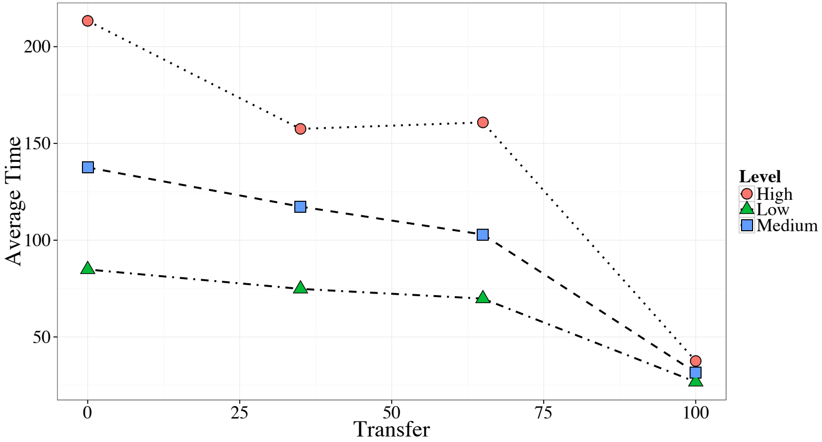
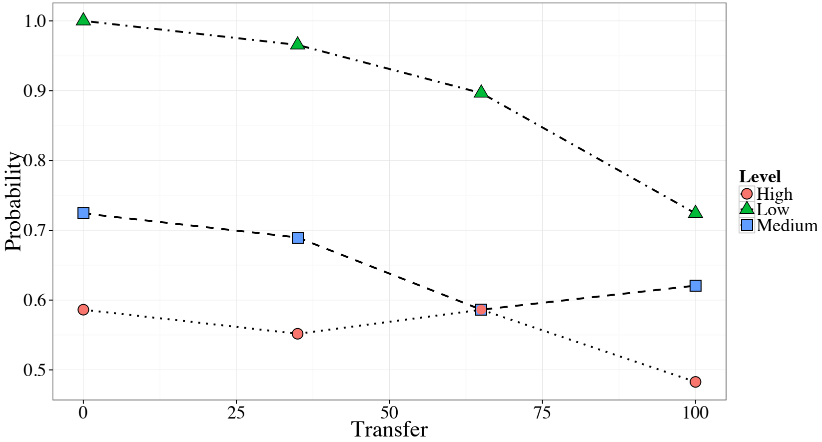


Time-Transfer Tradeoff



Probability-Transfer Tradeoff



Endogenous Transfers Results

Some Discussion

- There is a lot to learn about cost of attention
- The methodology can be used for various tasks
- The methodology can be used to ex-post classify complexity of mechanisms