Communication In Global Games: Theory and Experiment*

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Abstract

This paper introduces communication as a strategic choice in global games. To study the effects of communication, I consider four protocols (three one-round and one multi-round) and I characterize the resulting equilibria. The theory provides clear predictions, which are then tested in an experimental setting. Theoretically, all of the communication protocols studied in this paper equally improve welfare above that attainable without communication. This welfare improvement is achieved by reducing miscoordination and by allowing agents to select the payoff dominant as opposed to the risk-dominant equilibrium. The experimental results demonstrate that the multi-round protocol provides significantly higher welfare, while one-round communication has mixed effect.

JEL Classification: C71, C73, C92, D74;

Keywords: communication, global games, cheap-talk, coordination, experiment.

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1 Introduction

As a society, we consistently face situations where our actions need to be coordinated. Many of these settings are considered as a regime change game in which an existing status quo can be disrupted but only if sufficiently large number of individuals coordinate their actions against it. For example, oligarchic political regimes can be toppled but only if enough people join a protest. A currency can be devalued but only if sufficiently large number of individuals attack it. The coordination problem lying at the heart of these socio-economic phenomena gives rise to multiplicity of equilibria.

A well-established solution to this problem is to appeal to a structure known as global games, where there is breakdown in common knowledge and individuals have private information about the state of the world.[1] This paper builds on global games by introducing the possibility for individuals to communicate before taking an action. Such communication occurs in many circumstances. For example, in political crises, people take to social media to signal their intention to protest. During currency crises, banks many times issue statements about what their intentions are. Furthermore, in both cases, parties are not committed to their communicated intentions. In this paper, I introduce communication as a strategic choice between similarly informed participants in a global games setting. The implications of the theory are then tested in an experimental setting.

The classic game studied in the global games literature considers a coordination game involving two actions and two players, that suffers from multiplicity of equilibria under full information (Obstfeld (1986, 1996)). There are two pure strategy equilibria: payoff-dominant and risk dominant (Harsanyi and Selten (1988)). Carlsson and Van Damme (1993) introduce incomplete information to this setting so that individuals observe a noisy signal of the true state of the world. This perturbation provides a unique equilibrium selection.[2] Equilibrium in this game is characterized by a threshold strategy such that, for the signals above the cutoff, individuals choose the Pareto efficient equilibrium profile, and for signals below the cutoff, they choose a risk-dominant profile. Hence, under a natural assumption of imperfect information, global games methodology provides a unique-equilibrium selection.

Two types of inefficiencies are present in global games. First, individuals coordinate on the risk-dominant as opposed to the payoff-dominant equilibrium. Second, individuals may miscoordinate. The equilibrium induced by cheap-talk communication improves welfare by reducing both types of inefficiencies.

[1] Among many other papers, see Morris and Shin (1998, 2002) and Corsetti et al. (2004) for currency attacks, Goldstein and Pauzner (2005) and Rochet and Vives (2004) for bank runs, Dasgupta (2007) for delayed FDI investments, Corsetti et al. (2006) and Zwart (2007) for debt crises, Edmond (2013) for information manipulation by the regime, and Angeletos et al. (2007), Chassang (2010) and Mathevet and Steiner (2013) for a more dynamic setting. Heinemann et al. (2004, 2009) find experimental support for the theoretical predictions.

[2] Morris and Shin (1998, 2002) advance the work of Carlsson and Van Damme (1993) and apply the approach to currency attack settings.

The impact of communication on global games has to date been largely unexplored, both theoretically and experimentally. However, it was briefly discussed by Morris and Shin (2003), in a survey paper, in which the authors state: "[...] there cannot be a truthtelling equilibrium where the efficient equilibrium is achieved, although there may be equilibria with some partially revealing cheap talk that improves on the no cheap talk outcome." In this paper, I show that such partially revealing equilibrium exists and I characterize its properties. The paper first considers binary message space and studies its implications. Communication preserves the global games structure and improves welfare. Subsequently, a richer message space is introduced and the paper demonstrates that under the additional assumption of a noisy communication structure, only two types of messages are sent in equilibrium; hence, the equilibrium with richer message space is equivalent to the one with binary messages.[3]

The second part of this paper reports the results of an experiment that closely replicates the theoretical setting and tests the theoretical predictions. The experiment consists of five treatments: one *control* where no communication takes place, and four communication treatments, each testing a different communication protocol specified in the theory. In each treatment, the game is played between two subjects. In the control treatment, which replicates the baseline game without communication, subjects observe a private signal about the true state of the world and then make a decision between two alternatives. The remaining treatments introduce communication.

Communication can take many forms and it can be implemented in various ways. In the experiment, the first communication protocol follows an intuitive structure that comes from the equilibrium. In this treatment, called the *letter-messages* treatment, subjects are allowed to use two letters corresponding to their two possible intended actions. Here, since each letter could correspond to a different intended action, a common language (letters corresponding to actions) about the intended actions of players is easy to envision. To possibly allow subjects to convey more information over the intended action, the next treatment is introduced.

In the second treatment, called *number-messages* treatment, after subjects have observed their private signal (a number), they are able to send any number message to the other individual. In this treatment, the message space is the same as the signal space. Subjects need to find some common language using numbers to signal their intentions to the other subject. Though, this treatment allows for more information transmission (which should not happen in equilibrium), it is also harder without commonly understood messages. Hence, the *number-and-letter* treatment is introduced, which allows subjects to send both a number and a letter (intended action) message. While in equilibrium, the ability to send a letter message is redundant, the treatment is introduced because behaviorally, it might help clarify subject's planned actions.

[3] Certainly, as is common with many cheap-talk communication games, there are two types of equilibria: informative and uninformative (or babbling) equilibrium, where messages are ignored and the actions are the same as in the game without communication. This paper focuses on the informative equilibrium.

The final treatment is the *dynamic-cheap-talk* treatment, which is inspired by recent work demonstrating that real-time interaction can create an environment that facilitates extremely high levels of cooperation and coordination.[4] In this treatment, once subjects have observed their private signals, both make their initial choice (announce letter A or B corresponding to the actions available to them in the game) and have 20 seconds during which they can revise their choice at any instant. All the choices and changes are observable to both subjects, and the only payoff-relevant action is the last revision the subject makes before the 20 seconds are up.[5]

The experimental data demonstrates that the vast majority of the subjects use communication protocols to convey information. Moreover, in the three treatments where subjects can use letters corresponding to two alternatives, they use threshold strategies to transmit information. In the informative equilibrium (non-babbling) described in the theory section, following the information exchange, if individuals agree on an intended action, they should follow through with their initial intentions. The experimental data provides strong supports for this qualitative features of the equilibrium. In the experiment, if both subjects agree on an intended action, they follow through with their initial intentions in over 99% of the cases. However, the subjects set much more demanding cutoff levels than the theory predicts. The thresholds they use to send a message are too conservative and they are similar to the one they would use in the action stage in the absence of communication. Hence, they state what they would have done in the absence of communication, and they miss out on the welfare improvement through reduction of the threshold.

To analyze the behavior in the treatments with number-messages, the strategies are classified into four categories: two-partition, truth-telling, mixed, and babbling. Two-partition types find some common language to signal their intentions to play and account for 20% of this treatment. These types set low thresholds and make use of communication in the most effective way. Then, some subjects share their signals truthfully, and account for 40% of the number-message treatment. These types set action-stage cutoffs that are much higher than the optimal level. Moreover, the estimated threshold for truth-telling types is higher than the threshold in the control treatment with no communication. The subjects are classified into mixed types, if they truthfully report their signal for some realizations but partition or babble for others. Thresholds used by these types are similar to those of the truth-telling types. We observe low levels of babbling behavior—less than 4% of subjects send messages that seem to be unrelated to the underlying signals.

[4] See Oprea et al. (2011), Friedman and Oprea (2012), Deck and Nikiforakis (2012), Oprea et al. (2014), Bigoni et al. (2015), and Avoyan and Ramos (2017).

[5] This treatment is a modified version of the mechanism studied in Avoyan and Ramos (2017), where it provides high levels of efficient equilibrium selection in a coordination game with complete information. Avoyan and Ramos (2017) is an application of revision games (Kamada and Sugaya (2010), Calcagno et al. (2014)) to the minimum-effort game.

Although all communication treatments reduce miscoordination observed in the control treatment, the dynamic-cheap-talk treatment is the most effective communication protocol. This continuous interaction provides a significantly higher payoff compared to the baseline case. The aggregate effect of one-round information exchange on payoffs is not statistically significant and the impact is not universal for all types. The effect of multi-stage communication protocol is more general: the average payoff in this treatment is higher than in the control treatment; in addition, the empirical CDF of payoffs first-order stochastically dominates the one of the control treatment.

2 Related Literature

The effects of communication on global games has to date been largely unexplored. This paper introduces communication as a strategic choice between similarly informed participants and shows that communication impacts welfare in critical ways. The potential positive aspects of communication in global games was briefly discussed in Morris and Shin (2003). Apart from this discussion, communication in global games, if considered, is of "top-down" approach, where extra information is provided through a public signal, either directly or through a public choice.[6] Communication incentives in this paper differ from an environment where one policy maker communicates to all agents.

There are studies that examine the effects of communication in coordination games with incomplete information. Baliga and Morris (2002) study one-sided communication in a two player, two state game where the cost of taking a risky action can be high or low. One player is fully informed and can send a cheap-talk message to the other player who has a low cost of attacking. The authors show that full revelation of the cost type cannot be supported in equilibrium, similar to the intuition of communication in the Battle of the Sexes game as discussed in Banks and Calvert (1992).[7] Note, uncertainty is about the cost of taking an attack action for one of the players in contrast with the current paper where both players have incomplete information about the payoff relevant state of the world. While in the Baliga and Morris (2002), there is no communication in equilibrium (only babbling equilibrium exists), in the current paper there is informative equilibrium where messages convey information.[8]

While the theoretical literature on global games is vast, the experimental literature on the topic is more scarce.[9] Heinemann et al. (2004, 2009) experimentally study a speculative attack model under perfect and noisy private information, while Cabrales et al. (2007) test

[6] Hellwig (2002) was the first to introduce public signals to the model of Morris and Shin (1998) and characterize the equilibria in global games with public and private signals. For aggregation of information through prices or interest rates see Angeletos et al. (2006), Hellwig et al. (2006) and Ozdenoren and Yuan (2008). For information manipulation through biased media see Edmond (2013). Chen et al. (2016) model a rumor as a public signal.

[7] See also, Baliga and Sjöström (2004) and Baliga and Sjöström (2012), where the authors study an arms race game where the costs of acquiring new weapons are unknown to both players.

[8] The case discussed here is the positive slipover example since that case is the closest to the game in this paper.

[9] See Duffy (2008) for a survey of experimental work in macroeconomics.

the theory in a more discrete state space and two-players. These studies show that subjects' behavior is consistent with the theoretical predictions and the vast majority of subjects use threshold strategies. Cornand and Heinemann (2008) consider a combination of private and public signals, and in another treatment, two noisy public signals. one private and one public signal case provides higher probability of an attack compared to case with two noisy public signals. Subjects seem to overreact to the public signal when they also observe a private one. Similar results are found by Cornand and Heinemann (2014), where subjects overweight the public signal. Duffy and Ochs (2012) study a dynamic global game and find no significant differences between dynamic and static game thresholds.[10]

Qu (2013) experimentally studies the effect of endogenous information acquisition through market prices (see the theoretical model in Angeletos and Werning (2006)). An additional treatment introduced in the paper is cheap-talk protocol, which is similar to the intention-sharing treatment in the current paper. In Qu (2013), an experimenter acts as a mediator, collecting the intentions to attack and reports back to the group the percentage of subjects that have showed interest in investing. The experimental results provide evidence that informative equilibria are played and cheap-talk interaction improves coordination, which is in contrast to the results in this paper (the payoffs in intention-sharing treatment are statistically indistinguishable from the payoffs in the control treatment).

Szkup and Trevino (2012) experimentally study costly information acquisition model introduced in Szkup and Trevino (2015). The authors show that subject behavior is consistent with the theoretical predictions of a threshold strategy usage; however, in the information acquisition phase, subjects invest too much in the precision. The experiment in this paper adds communication stages to the base game of Szkup and Trevino (2012), keeping all relevant parameters the same. This allows comparisons of the control treatment in the current paper with the control treatment in their paper.[11]

This paper is related to studies on costly information acquisition.[12] The main difference between the current paper and this literature is that once the cost of acquiring information is not present (we have a cheap-talk game) and we let players choose what type of information to share (communication is strategic), interim incentives and an assumption of residual uncertainty eliminate full-revelation which may result in multiplicity of equilibria. In the equilibrium of some of these environments, all players have the same information, which leads to a common posterior and, in turn, to a multiplicity of equilibria. In this paper, there are no multiplicity issues in the action stage since players only reveal part of their information—there is,

[10] Shurchkov (2013) tests a two-period version of the model in Angeletos et al. (2007) and provides support for most of theoretical predictions. The experimental results indicate that knowledge about the survival of the status quo in the first stage discourages the attack in the second stage.

[11] See Trevino (2017) for a two country model of contagion.

[12] See, for example, Yang (2015), Szkup and Trevino (2015), Denti (2017) and in linear best-response games see Hellwig and Veldkamp (2009), Myatt and Wallace (2011), Colombo et al. (2014), Pavan (2014). See also Mahdavifar et al. (2017), where the authors consider non-strategic noisy information sharing.

however, multiplicity in the communication stage.

An extensive experimental literature studies the effects of communication in coordination games with complete information and demonstrates that cheap talk can facilitate coordination on an efficient equilibrium (for a critical survey see Devetag and Ortmann (2007)). Van Huyck et al. (1990) showed a strong tendency of play to diverge towards inefficient risk-dominant equilibrium in the minimum-effort game, which prompted a vast literature on the issue. Cooper et al. (1992) find that with one-way communication the payoff-dominant equilibrium is chosen more often in a 2×2 Stag and Hunt game, but two-way communication does so to a greater extent. Blume and Ortmann (2007) test the effect of cheap talk communication about actions both in the minimum-effort and median games. They find that messages facilitate high rates of convergence to the Pareto-dominant equilibrium.[13] In contrast with this literature, in this paper, similar one-round communication treatments fail to significantly improve welfare. This result might be due to the fact that coordination games with incomplete information have more layers of difficulty, since the messages provide the intentions to play and some information on the underlying state.

3 Model

The section first introduces the baseline game without any communication. Subsequently information sharing protocols are considered. The framework for the underlying game is similar to the 2×2 model of global games introduced by Carlsson and Van Damme (1993) and further advanced by Morris and Shin (1998, 2002).[14]

3.1 The Baseline Framework without Communication

The state of the world is characterized by a fundamental $\theta \in \Theta$. In the currency attack example, θ describes the net gain from a devaluation, and in the regime overturning example, it describes the strength of the government. Players, indexed by $i \in I = \{1, 2\}$, are ex-ante identical and simultaneously choose between two actions: they can either attack the status quo ($a_i = 1$) or refrain from attacking ($a_i = 0$). Thus, the action space for player i is $A_i = \{0, 1\}$. Attacking has a fixed cost of $c > 0$, which could be interpreted as a direct transaction cost, or simply the opportunity cost. The fundamental θ is normalized so that the attack is successful if and only if $\theta > k(\theta)$, where $k(\theta)$ is a critical mass of players needed for a successful attack under the state of the world θ. We let $\underline{\theta} := k^{-1}(2)$ and $\bar{\theta} := k^{-1}(1)$. A player's incentive to attack increases with the aggregate size of an attack; hence, players' actions a_i are strategic complements.

All players start with a common prior for θ, they believe θ is drawn from a normal distri-

[13] See also Berninghaus and Ehrhart (2001), Burton and Sefton (2004), Devetag (2005), Charness (2000), Brandts and Cooper (2006), and Chaudhuri et al. (2009).

[14] The action stage without communication is in line with the second stage of the model studied in Szkup and Trevino (2012).

bution with mean θ_0 and variance σ_θ^2.[15] Each player i receives a private signal $x_i = \theta + \sigma_i \varepsilon_i$, where $x_i \in X_i$ and $\varepsilon_i \sim \mathcal{N}(0,1)$ is a noise, independent and identically distributed across players and independent of θ. Given the realization of player i's signal, x_i, the posterior distribution of θ is normally distributed with mean $\tilde{\theta}_i$ and variance $\tilde{\sigma}_i^2$, where $\tilde{\theta} = \frac{x_i \sigma_\theta^2 + \theta_0 \sigma_i^2}{\sigma_\theta^2 + \sigma_i^2}$ and $\tilde{\sigma}_i^2 = \frac{\sigma_\theta^2 \sigma_i^2}{\sigma_\theta^2 + \sigma_i^2}$.

Player i's action strategy is $a_i : X_i \to A_i$ and player i's utility is $u_i : A \times \Theta \to \mathbb{R}$, where $A = A_i \times A_j$ and

$$u_i(a;\theta) = (\mathbb{1}_{\{\theta > k(\theta)\}} \theta - c) a_i.$$

The payoffs can be summarized in a matrix form, see Figure 1:

	Success	*Failure*
Attack	$\theta - c$	$-c$
Not Attack	0	0

Figure 1: Payoff Matrix

The game with common knowledge of the state of the fundamental θ (complete information game) serves as an intuitive baseline to the game with private information. For $\theta < \underline{\theta}$, the devaluation will not happen even if both players attack; hence, the dominant strategy is to refrain from attacking and to keep the status quo. If $\theta \geq \overline{\theta}$, one player choosing to attack is sufficient to abandon the status quo; hence, the dominant strategy is to attack. The case of interest is when $\theta \in [\underline{\theta}, \bar{\theta})$, where two pure-strategy equilibria are sustainable: (i) all players attack and the status quo is abandoned; and (ii) all players refrain from attacking and the status quo is preserved.

Carlsson and Van Damme (1993) have shown that the multiplicity of equilibria described above is due to complete information of the payoff function. If players don't observe the true value θ but rather a noisy private signal of it, then there is a unique equilibrium. This equilibrium is characterized by a symmetric threshold strategy such that player i attacks the status quo if and only if the signal realization is greater than the threshold x_{NC}^*; that is the players $i \in \{1,2\}$ follow a symmetric threshold strategy

$$a_i(x_i) = \begin{cases} 1, & \text{iff } x_i \geq x_{NC}^* \\ 0, & \text{iff } x_i < x_{NC}^* \end{cases}$$

For completeness of the baseline framework, this section is finished by solving for the latent threshold x_{NC}^*. Let $g(\theta, x_j^*)$ be player i's payoff given θ and the other player's threshold x_j^*. Player i's expected payoff conditional on taking an attack action is (details are in Appendix

[15] Alternatively, we could have assumed an improper uniform prior for θ on $\mathbb{R}$ and common public signal.

A.1):

$$\mathbb{E}[g(\theta, x_j^*)|x_i] = \int_{\underline{\theta}}^{\overline{\theta}} \theta \left[\Pr(x_j \geq x_j^*|x_i, \theta)\right] p(\theta|x_i) d\theta + \int_{\overline{\theta}}^{\infty} \theta p(\theta|x_i) d\theta - c$$

Player i will choose to attack the status quo if, and only if, the expected payoff is greater than zero, $\mathbb{E}[g(\theta, x_j^*)|x_i] \geq 0$. To solve for an optimal threshold x_{NC}^*, all we need is to find a signal for which player i is indifferent between attacking the status quo and refraining from attacking, that is $\mathbb{E}[g(\theta, x_j^*)|x_{NC}^*] = 0$, given the optimal threshold of player j, x_j^*. There exists a unique, dominance solvable equilibrium in which both players use threshold strategies with cutoff x_{NC}^*.

3.2 Cheap Talk Communication

In this section, we introduce communication in the form of binary signals where the message space of player i is $M_i = \{0, 1\}$. The case of richer message spaces, $M_i = X_i$ and $M_i = X_i \cup \{0, 1\}$, is discussed later in the section. Under an assumption of residual uncertainty, the richer message space cases reduce to binary message setting; hence, the focus of this section is characterizing the equilibria with two messages.

Figure 2: The Timing of the Game

Once player i has observed their private signal $x_i \in X_i$ they send a message $m_i : X_i \to M_i$ to the other player before deciding to either attack the status quo or refrain from attacking, $a_i : X_i \times M \to A_i$, $M = M_i \times M_j$. All messages are sent and received simultaneously and sending a message bears no cost. The timing of the whole game is given in Figure 2.

This game has two types of communication equilibria: (i) informative equilibrium, a two partition equilibrium in which there is pooling into two types, say "intention to attack" and "intention not to attack"; and (ii) uninformative equilibrium, babbling equilibrium in which messages are ignored and the whole game reduces to the baseline framework without communication as in Section 3.1. One can interpret this pooling into two types as players sharing their intentions to get involved or not, so that receiving a message $m_j = 1$ or $m_j = 0$ is interpreted as "I plan to attack" or "I do not plan to attack."

Below I present the main theorem, its implications and then analyze welfare. Section 4 and Appendix A.3 contain the solution of the model, all required results, and discussions.

Theorem 1 *There is symmetric pure strategy perfect Bayesian equilibrium. The equilibrium is monotone in the sense that there exist thresholds* $(x_C^*, \bar{x}^*)$ *such that in the communication stage player* i *sends a message* $m_i(x_i)$ *and in the action stage player* i *takes an action* $a_i(x_i; x_C^*, \mathcal{I})$*, where*

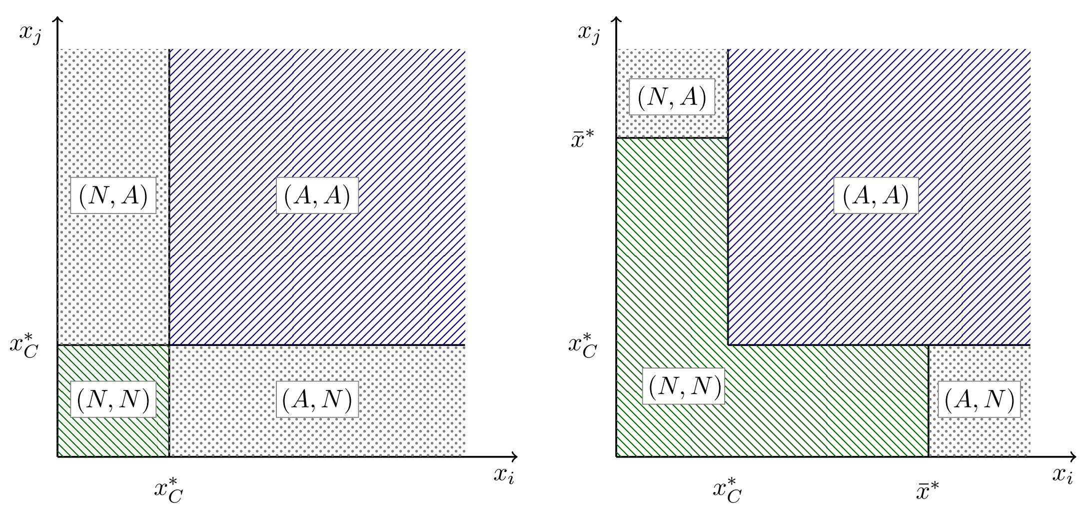
$$m_i(x_i) = \begin{cases} 1, & \text{if } x_i \geq x_C^* \\ 0, & \text{if } x_i < x_C^* \end{cases} \tag{1}$$

$$a_i(x_i; x_C, \mathcal{I}) = \begin{cases} 1, & \text{if } x_i, x_j \geq x_C \text{ or } x_i \geq \bar{x}^* \\ 0, & o.w. \end{cases} \tag{2}$$

$\mathcal{I} = (m_i, m_j)$*, for* $x_i \in X_i$ *and* $i \in I$*,* $i \neq j$*.*

Let us look at the outcomes of communication under the partially informative equilibrium for all combinations of signal realizations $(x_i, x_j) \in \mathbb{R}^2$. Figure 3 summarizes the messages and the actions of Theorem 1. If both players receive signals below threshold x_C^*, then they send a message not to attack and then both abstain from attacking in the action stage and keep the status quo. Similarly, if both players receive signals above threshold x_C^*, then players send a message to attack and they both attack. Thus, if players agree on an intended action, they follow through with their initial intentions.

Figure 3: Informative Equilibrium Messages and Actions



(a) Messages Sent

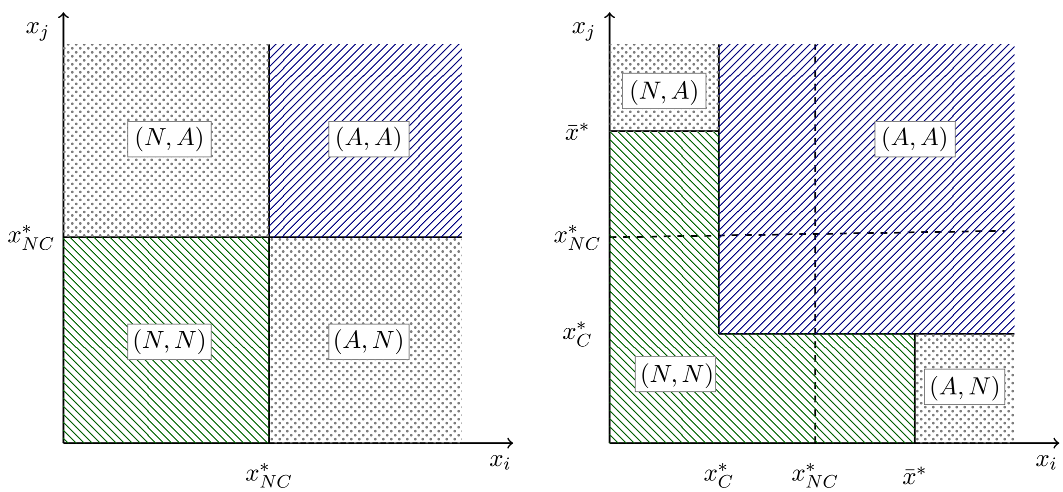
(b) Actions Taken

If the intended actions disagree, players use a significantly more demanding cutoff. Consider the case when realized signals are in the gray areas of Figure 3a, area (A, N) and (N, A) represents the case where one player has signal greater than x_C^*, while the other has a signal less than x_C^*. Depending on how high the attack signal is, a player who received a no attack message might still decide to unilaterally take on the status quo. In particular, player i attacks the status quo if their realized signal x_i is greater than threshold $\bar{x}^*$.

Notice that if a player's message conveys an intention not to attack, there is no way to persuade them to switch and attack in the action stage. The intuition behind the statement is that if a player has information under which he would choose to attack if the other player were to attack, then this player would have sent an attack message ("A"). This is because, an attack message (weakly) increases the probability of the other player following and the expected payoff is higher. Hence, the communication threshold x^*_C is based on the most optimistic case where the other individual has positive information and is going to attack.

To evaluate the welfare effects of communication under the informative equilibrium, Figure 4 presents the equilibrium outcomes with and without communication. In the left panel: after receiving private signals, x_i and x_j, player i and player j take an attack action if, and only if, their private signal is greater than x^*_{NC}. Since there is no communication, the actions are based solely on own private signals. If both signals are greater than x^*_{NC}, both players attack and successfully abandon the status quo. Similarly, if both signals are less than x^*_{NC}, both players choose not to attack and the status quo remains in place. If only one player attacks and the other does not, we get mismatched actions, the gray regions of the left panel.

Figure 4: Equilibria Outcomes without and with Communication



(a) Without Communication

(b) With Communication

The right panel of Figure 4 presents the equilibrium outcomes of the game with communication. There are two main types of improvements arising from communication: (i) switches to (A, A) from (N, N); and (ii) increased coordination from switches from (A, N) and (N, A). For a sizable range of the inefficient selection in the baseline framework, the equilibrium switches from risk-dominant to payoff-dominant. That is, if realized signals were in the region $[x^*_C, x^*_{NC}) \times [x^*_C, x^*_{NC})$, then, without communication the outcome would be (N, N). However, with communication the outcome is (A, A). Let us focus on the area

$[x^*_{NC}, \infty) \times [0, x^*_{NC})$ where, without communication the outcome is (A, N). Adding communication divides this area into three regions. Communication enables coordination on attack-action and not-attack action, where there used to be mismatched actions without communication. Notice, the area $[\bar{x}^*, \infty) \times [0, x^*_C)$ remains as miscoordination. The next section provides more details on the quantitative gains of these changes.

4 Solving the Model

In this section we solve the model and give more details on the main theorem stated in Section 3.2. There is an equilibrium under which messages are ignored and we get babbling in the communication stage and the baseline framework in the action stage. The question is whether the informative equilibria of the game exists.

The outline of the proof is as follows. First, the communication strategy is a threshold rule. Then, the paper develops a way to update the prior using a combination of two types of signals. Consequently, we get an action stage which is similar to the standard global games. Under an assumption on the ratio of private and public signals, we get a unique solution in the action stage.

The following is a definition of the equilibrium of the whole game that includes the communication and action stages.

Definition 1 *Communication strategy x^*_C, action strategy a^*, and belief rule p, constitute a pure strategy symmetric perfect Bayes equilibrium if*

[i] For any $x_i \in X_i$,

$$x^*_C(x_i) \in \arg\max_{m \in M_i} \int_{\theta \in \Theta} u(a(x_i; (m(x_i), m_{-i}), a_{-i}; \theta) p(\theta | x_i, \mathcal{I}) d\theta$$

[ii] For a given $m \in M_i$,

$$a^*(x_i; \mathcal{I}) \in \operatorname{argmax}_{a \in A_i} \int_{\theta \in \Theta} u(a, a_{-i}; \theta) p(\theta | x_i, \mathcal{I}) d\theta$$

[iii] p is obtained by Bayes rule

$$p(\theta | x_i, \mathcal{I}) = \frac{p(x_i, \mathcal{I} | \theta) p(\theta)}{\int_{\Theta} p(x_i, \mathcal{I} | \theta) p(\theta) d\theta},$$

where

$$m(x_i) = \begin{cases} 1, & \text{if } x_i \geq x_C \\ 0, & \text{o.w.} \end{cases} \tag{3}$$

and

$$a(x_i;\mathcal{I}) = \begin{cases} 1, \text{ if } (x_i \geq x_C \wedge x_j \geq x_C) \vee (x_i \geq \bar{x}) \\ 0, \text{ o.w.} \end{cases} \tag{4}$$

$\mathcal{I} \in M_i \times M_j, \mathcal{I}_1 = (m, 1), \mathcal{I}_0 = (m, 0)$ *and* $i \in I, i \neq j$.

Lemma 1 *The communication strategy* $m_i : \Theta \to M_i$ *is a threshold rule.*

Proof. See Appendix A.3. ∎

Given Lemma 1, consider the action stage of the game and note that for different communication stage equilibria the posterior distribution will be different. In particular, in the case of babbling equilibrium, since there is no learning from the messages, the posterior distribution will coincide with the one in the baseline framework of Section 3.1. The case of partially informative communication stage is more involved, and it is discussed below, but more details are in Appendix A.3.2.

Player i's information set is (x_i, m_j), where $x_i|\theta \sim N(\theta, \sigma_i^2)$ and $m_j|\theta \sim \text{Bern}(1 - q(\theta))$ with $q(\theta; x_C^*, \sigma_j^2) = \Phi(x_C^*; \theta, \sigma_j^2)$. Combining continuous and binary signals with a prior on θ, results in the following result (see Appendix A.3.2).

Lemma 2 *If the prior for* θ *is* $N(\theta_0, \sigma_\theta^2)$*, then the posterior distribution of* θ *is Extended Skew-Normal, with density*

$$p(\theta|x_i, m_j) = \frac{1}{\Phi(\tau_c)} \frac{1}{\omega_c} \phi\left(\frac{\theta - \xi_c}{\omega_c}\right) \Phi\left(\tau_c \sqrt{1 + \alpha_c^2} + \alpha_c \frac{\theta - \xi_c}{\omega_c}\right)$$

where

$$\xi_c = \frac{\sigma_i^2 \theta_0 + \sigma_\theta^2 x_i}{\sigma_i^2 + \sigma_\theta^2}, \quad \omega_c^2 = \frac{\sigma_i^2 \sigma_\theta^2}{\sigma_i^2 + \sigma_\theta^2}$$

and

$$\alpha_c = \frac{\alpha}{\sqrt{1 + \sigma_i^2/\sigma_\theta^2}}$$

$$\tau_c = \tau \sqrt{\frac{1 + \alpha^2}{1 + \alpha_c^2}} + \frac{\alpha(\theta_0 - x_i)}{\sigma_i (1 + \sigma_\theta^2/\sigma_i^2) \sqrt{1 + \alpha_c^2}}$$

In abbreviated form, this is written $ESN(\xi_c, \omega_c, \alpha_c, \tau_c)$.

For any distribution $ESN(\xi_c, \omega_c, \alpha_c, \tau_c)$, ξ_c is referred to as the location parameter, ω_c is the scale parameter, α_c is the slant parameter, and truncation parameter τ_c (Azzalini (2013)). Using Lemma 2, the posterior mean and variance are given by the following expressions (see A.3.2 for derivation):

$$\mu = \xi_c + \frac{\phi(\tau_c)}{\Phi(\tau_c)}(\delta_c \omega_c), \quad \sigma^2 = \omega_c^2 \left(1 - \delta_c^2 \frac{\phi(\tau_c)}{\Phi(\tau_c)} \left[\tau_c + \frac{\phi(\tau_c)}{\Phi(\tau_c)}\right]\right)$$

respectively, where $\delta_c := \frac{\alpha_c}{\sqrt{1+\alpha_c^2}}$. Note that if $\delta_c = 0$, then the mean μ is the location parameter ξ_c and the variance σ^2 is the scale parameter ω_c^2.

The action stage is similar to the baseline game with the difference that players have ESN posteriors. As indicated by Vives (2005) and Van Zandt and Vives (2007), global games belong to the class of supermodular games and the equilibrium selected in the perturbed game is the Harsanyi and Selten (1988) risk-dominant one. The result of Van Zandt and Vives (2007) provides the existence of greatest and least Bayesian Nash equilibria and these are monotone in type (see Appendix A.2).[16] First, let us find the conditions that will provide the least and greatest Bayesian Nash Equilibria in monotone strategies and then prove uniqueness by showing that these two equilibria coincide.

Conditional on the other player's message ($m_j = 0$ or $m_j = 1$) and communication threshold x_C^*, we assume that players follow a symmetric threshold strategy

$$a_i(x_i; x_C^*, \mathcal{I}) = \begin{cases} 1, \text{ if } x_i \geq x^*(\mathcal{I}) \\ 0, \text{ if } x_i < x^*(\mathcal{I}) \end{cases}$$

where $\mathcal{I} = (m_i, m_j)$. Based on whether $m_j = 0$ or $m_j = 1$, $x^*(\mathcal{I})$ will be different. Hence, there are two thresholds: the other player sent "attack" message, call it $\underline{x}^*$ and the other player sent "no attack" message, call it $\bar{x}^*$.

Equation 5 provides the expected payoff in the action stage for a player i choosing to attack conditional on information $(x_i, x_C^*, \mathcal{I})$. In addition, player i assumes that player j follows a threshold strategy $x_j^*(\mathcal{I})$.

$$V_a(x_i, x_j^*; x_C^*, \mathcal{I}) = \int_{\underline{\theta}}^{\overline{\theta}} \theta \left[\Pr(x_j \geq x_j^* | \theta, x_i, x_C^*, \mathcal{I}) \right] p(\theta | x_i, x_C^*, \mathcal{I}) d\theta + \int_{\overline{\theta}}^{\infty} \theta p(\theta | x_i, x_C^*, \mathcal{I}) d\theta - c \quad (5)$$

where

$$p(\theta | x_i, x_C^*, \mathcal{I}) = \frac{p(x_i, x_C^*, \mathcal{I} | \theta) p(\theta)}{\int_{\Theta} p(x_i, x_C^*, \mathcal{I} | \theta) p(\theta) d\theta} \quad (6)$$

$\mathcal{I} \in M_i \times M_j$, $x_i \in X_i$ and $i \in I$, $i \neq j$. Next, the equilibrium of the action stage is defined.

Definition 2 *Given messages $m = (m_i, m_j)$ and message thresholds $x_C^* = (m_i^*, m_j^*)$ from the communication stage, an equilibrium in monotone strategies for action stage of the game is a pure strategy profile $\boldsymbol{a}^* = (a_i^*, a_j^*)$ and corresponding thresholds $\boldsymbol{x}^* = (x_i^*, x_j^*)$ such that x_i^* solves*

$$V_a(x_i^*, x_j^*; \mathcal{I}) = 0,$$

[16] One of the conditions, boundedness of the utility function is violated since $u(\theta) \to \infty$ when $\theta \to \infty$. However, Szkup and Trevino (2012) extend Van Zandt and Vives (2007) result for unbounded utility functions that are still integrable by adding further assumptions, see online Appendix B for more details.

where

$$a_i^*(x_i; x_C^*, \mathcal{I}) = \begin{cases} 1, \text{ if } x_i \geq x_i^*(\mathcal{I}) \\ 0, \text{ if } x_i < x_i^*(\mathcal{I}) \end{cases}$$

for all $i \in I, \ i \neq j$.

Conditional on the case of $m_j = 1$ or $m_j = 0$, $\underline{x}^* := x_i^*(\mathcal{I}_1)$ and $\bar{x}^* := x_i^*(\mathcal{I}_0)$ solve

$$V_a(\underline{x}^*, \underline{x}^*; x_C^*, \mathcal{I}_1) = 0 \text{ and } V_a(\bar{x}^*, \bar{x}^*; x_C^*, \mathcal{I}_0) = 0,$$

where $\mathcal{I}_1 = (\cdot, 1)$ and $\mathcal{I}_0 = (\cdot, 0)$. The expected payoff of attack action with realized signal x_i, conditional on $m_j = 1$, $a_j = 1$ and $m_j = 0$, $a_j = 0$, can be written as

$$V_1 = \int_{\underline{\theta}}^{\infty} \theta p(\theta | x_i, x_i, \mathcal{I}_1) d\theta - c \tag{7}$$

$$V_0 = \int_{\bar{\theta}}^{\infty} \theta p(\theta | x_i, x_i, \mathcal{I}_0) d\theta - c \tag{8}$$

where $\mathcal{I}_1 = (\cdot, 1)$ and $\mathcal{I}_0 = (\cdot, 0)$. Observe that both equations, (7) and (8), are bounded from below by $-c$. In addition, recall that the utility function $u_i(a; \theta)$ is not bounded from above. Using Lemma 6 we get that V_1 and V_0 are increasing in x_i, and therefore, we have a single crossing for each case.

Similar to the literature on global games, there exists a unique solution of the action stage of the game given a condition on the relative informativeness of the private signal compared to the public signal. If σ_i/σ_θ is sufficiently small, that is, the private signal is sufficiently more precise than the public signal, then we get the following proposition.

Proposition 1 *There exists a unique, dominance solvable equilibrium of the actions stage of the game in which player* $i \in I$ *uses threshold strategies, characterized by* $(\underline{x}^*, \bar{x}^*)$, *if* $\gamma(\sigma_\theta, \sigma_i) > \sqrt{2}$.

Proof. See Appendix A.3.4. ■

If player j's message conveys an intention to attack, then player i takes an attack action if his private signal is greater than $\underline{x}^*$. If player j 's message states an intention to abstain from attacking, player i is not able to persuade player j to switch and attack in the action stage. However, even if player j is not attacking, player i might still decide to challenge the status quo. If the private signal is greater than the cutoff $\bar{x}^*$ then player i takes an attack action, as he believes the state of the world to be in the region where one player suffices to successfully overturn the status quo.

5 Richer Message Spaces and Noisy Cheap Talk

In the Section 3.2, intermediate communication stage with binary messages is introduced and its welfare benefits are established. This section examines whether enriching message space can provide even higher benefits than two message case since the signal space is itself rich (the whole real line). When the message space is the signal space there exists a fully revealing equilibrium of the communication stage. However, this equilibrium induces common posterior of the state of the world, which breaks the global games structure of the action stage and reintroduces multiplicity. Moreover, fully reveling equilibrium is not robust to small residual uncertainty. The second part of this section analyses any finite partition equilibria and discusses its payoff equivalence to the two-partition equilibrium.

Suppose the messages sent are the signal realizations $m_i = x_i$ and messages received are taken at face value. This communication stage induces common posterior $\theta | x_i, m_j \sim \mathcal{N}\left(\check{\theta}, \check{\sigma}^2\right)$. To calculate $\check{\theta}, \check{\sigma}^2$, let the average of the two signals be $\bar{x} := \frac{1}{2}(x_1 + x_2)$. Since the average signal is a sufficient statistic, we will refer to it as the player i's combined signal. Using the standard approach, the prior belief is updated with the combined signal $\bar{x}$, which induces a common posterior $\theta | \bar{x} \sim \mathcal{N}\left(\check{\theta}, \check{\sigma}^2\right)$, where $\check{\theta} = \frac{\bar{x}\sigma_\theta^2 + \theta_0 \sigma^2}{\sigma_\theta^2 + \sigma^2}$, $\check{\sigma}^2 = \frac{\sigma_\theta^2 \sigma^2}{\sigma_\theta^2 + \sigma^2}$ and $\sigma^2 := \frac{1}{4}(\sigma_1^2 + \sigma_2^2)$.

Proposition 2 *Consider any message space M_i:*
a) Any finite partition equilibria is payoff equivalent to the unique two-partition equilibrium;
b) There is no fully revealing equilibrium with message distortion ξ.

Fully revealing messages result in common posterior since all players have the same information. This environment with common posterior induces similar multiplicity that is present in the game with complete knowledge of the state of the fundamental θ. That is, after communication stage, both players taking an attack action and both players abstaining from attack is an equilibrium. The first-best outcome of the game is for both players to take an attack action, if posterior mean, $\check{\theta}$, is greater than the cost of attacking c. Note, player weakly prefers the other player to always attack. If there is no noise, fully revealing the signals is an equilibrium, since once signals are combined, players' preferences are perfectly aligned. However, suppose there is some residual uncertainty, that is, for example, let messages get distorted in the receiving process with noise ξ that is independent of the state and signals. Then, player i would exaggerate the signal just to make sure that the other player attacks. This slight misalignment distorts the fully revealing equilibrium.

Consider any partition equilibria in which players partially reveal their private information. If a message is payoff relevant, that is for a given signal x_i, the probability of player i taking an attack action is strictly positive for some message $m_j \in M_j$, then player i would want to send a message that induces the other player to attack with the highest probability. However, if player i's signal is such that he would never attack, then messages are payoff irrelevant and he will send a no-attack message (see Appendix A.3 for the discussion on this indifference issue). Thus, any finite partition equilibria is payoff equivalent to a two-partition equilibrium, where messages are interpreted as an intention to attack or not.

6 Experimental design

The experimental sessions were conducted at the Center for Experimental Social Science (CESS) laboratory at New York University (NYU), using the software z-Tree (Fischbacher (2007)). Subjects were recruited from the general population of NYU students. The experiment lasted about 50 minutes and, on average, subjects earned $\$21$ that included a $\$10$ dollar show-up fee. In each session, written instructions were distributed to the subjects and also read aloud.

The experiment consists of five different treatments. In each treatment, participants are randomly divided into pairs. These assignments are fixed for the duration of the experiment, which consists of 50 independent and identical rounds. In each round, all the subjects have to choose between two alternatives, A and B. Before the subjects make their A/B choice they receive a private signal about the true state X, which is a random variable that affects both players' payoff structure. The parameters used are similar to the ones in Szkup and Trevino (2012). In particular, the control treatment of this study as described below coincides with the direct-choice control treatment with exogenous precision level 4 in Szkup and Trevino (2012).

In all of this experiment's treatments, the true state X is randomly drawn from a normal distribution with mean 50 and standard deviation 50. This randomization is done once and it is used in every treatment. This choice ensures the differences between treatments are due to the changes in communication protocols and not the precedents caused by the particular order of realized values of X. The coordination region for which two pure-strategy equilibria are sustainable under complete information is $[0, 100)$. All subjects receive a private signal randomly drawn from a normal distribution that is centered at the true value of X and has standard deviation of 10. Choosing action A always bears a cost of 18 points, making the coordination interval effectively $[18, 100)$.

The first treatment in the experiment is the *control* treatment, T_0, where the subjects observe their private signal and then proceed to make their A/B choice. Once both subjects in the same pair have made their selection, the round is over and they receive feedback. The subjects observe the realization of X in this round, their own private signal realization, the choice made by the subject and the other pair member, and the individual payoff in the round. Note that no communication or any sort of interaction allowed in the control treatment. All the other treatments in the experiment involve some type of communication component.

In the *signal-sharing* treatment, T_S, once the subjects have observed their private signals about the true state X, but before they make their final decisions, they can send a message that can be any number $m_n \in \mathbb{R}$. These messages can be interpreted as "My signal is __."

The message-sending stage is simultaneous, and once both subjects in the same pair receive the other's message, they can proceed and decide between alternatives A and B. The round is over when both subjects make their A/B choice. According to the theoretical results, the informative equilibrium in this paper requires information withholding; subjects should pool themselves into two types and send signals accordingly.

A vast experimental literature observes over-communication and truth-telling in experiments on strategic information-transmission games. Some studies attribute truth-telling to the intrinsic cost of lying, or claim ethical types exist who would never lie for economic gain. Without identifying the source of over-communication, we introduce an intention-sharing treatment to mitigate these forces. In the *intention-sharing* treatment, T_I, once the subjects have observed their private signals but before they make their final decisions, they can send a message that can only be a letter $m_L \in \{A, B\}$. These messages can be interpreted as "I'm going to choose the alternative __ ." Note that while the theoretical predictions of this treatment are similar to the one for treatment T_S, we have reduced the difficulty by providing a common language about intentions.

We introduce the third communication protocol to test whether expanding the message space to sharing the intended action and their signals could help aid higher levels of coordination. In the *intention-and-signal-sharing* treatment, $\mathrm{T}_{I\&S}$, once the subjects have observed their private signals but before they make their final decisions, they can send a message that can be any number $m_n \in \mathbb{R}$ and a letter $m_l \in \{A, B\}$. These messages can be interpreted as "My signal is __" and "I'm going to choose the alternative __ ." Providing a letter message in addition to the number message is theoretically redundant. In equilibrium, when a player sends a number message, what that number implies in terms of intended actions is common knowledge. However, experimentally, the effect is not clear.

Our final treatment is dynamic cheap talk about the intentions to play. The *dynamic-cheap-talk* treatment, T_{DCT}, is a modified version of the revision mechanism studied in Avoyan and Ramos (2017).[17] The mechanism Avoyan and Ramos (2017) applied to the setting in this paper is as follows. Once the subjects have observed their private signals, both of them make their initial A/B choice. Players see what the pair member's initial choice is, and they have 20 seconds during which they can revise their initial choice at any instant. All the choices and changes are observable for both players in the pair. The only payoff-relevant action is the last revision the subject makes before the 20 seconds are up. That is, all choices and changes up until the 20th second are not payoff relevant. The information about both subjects' choices and all the revisions are represented in a graph for easier access; see Figure 11 in Appendix B (the graph is similar to the one in Avoyan and Ramos (2017); the main difference is that in the revision stage in the treatment T_{DCT}, subjects can change their choice at any instant, whereas

[17] Avoyan and Ramos (2017) apply the mechanism to the minimum-effort game and they argue that this mechanism drastically decreases the strategic uncertainty, leading players to coordinate on the most efficient equilibrium.

in Avoyan and Ramos (2017) the messages are sticky and revisions are only possible when a revision opportunity is randomly awarded to a subject). The round is over when 20 seconds run out.

Treatments	Communication	Message Space
T_0	None	—
T_S	Cheap Talk	Signals
T_I	Cheap Talk	Actions
$T_{I\&S}$	Cheap Talk	Signals and Actions
T_{DCT}	Dynamic CT	Actions

Table 1: Experimental Design

In all the communication treatments, T_S, T_I, $T_{I\&S}$ and T_{DCT}, the end-of-the-round feedback consists of the realized value of X, the subject's own signal realization, the message sent and the message received, the choice made by the subject and the other pair member, and the individual payoff in the round. After 50 rounds, subjects take a short survey and they receive their final payment that includes the show-up fee and the average of five rounds of the payoffs randomly chosen from all 50 rounds (survey results are summarized in Appendix B.3).

Table 1 summarizes the experiment treatments, communication protocols, and the message space available to the subjects.

6.1 Numerical Example

Consider the game with parameters as implemented in the experiment described in Section 6. For this example, we can calculate Type I and Type II errors and quantify the gains of communication.

The state of the world is governed by a normal distribution with mean 50 and standard deviation 50. The coordination region is $[0, 100)$, that is, if $\theta < 0$, attack can not be successful even if both players attack. If $\theta \geq 100$, one player is sufficient to induce successful attack and receive $\theta - c$, regardless of what the other player does. Choosing an attack action bears a cost of 18 points. All subjects receive a private signal randomly drawn from a normal distribution that is centered at θ and has standard deviation of 10.

Without communication, the game has a unique equilibrium which is monotone in type and it is characterized by a threshold $x^*_{NC} = 28.31$. With communication, message sending threshold is $x^*_C = 11.47$. Conditional on receiving an attack message, the action threshold is $\underline{x}^* = 11.47$, while the threshold for attacking conditional on receiving no-attack message is $\bar{x}^* = 178.24$. If we combine two signals and follow the best outcome, then both player should attack if the average of the two signals is greater than $x_{FB} = 17.36$.[18]

[18] For $\breve{\theta} = \frac{\bar{x}\sigma_\theta^2 + \theta_0\sigma^2}{\sigma_\theta^2 + \sigma^2} \geq 18$, we need $\bar{x} \geq 17.36$.

	No Communication	Communication	First-best
Type I Error	4.4%	2.4%	1.6%
Type II Error	9.8%	2.0%	1.8%

Table 2: Numerical Comparison

Table 2 presents the results of Type I and Type II errors for three cases. The data is simulated using the parameters above and calculations use the corresponding equilibria. Without communication, in 9.8% of the time, players would miss out on attacking while successful attack would have been possible, while with communication this type of error is reduced to 2%. Similarly, false attacks for which players pay the cost of attacking but receive no gain, is reduced from 4.4% to 2.4%.

7 Experimental Results

Testable predictions provided by the theory are used to guide our analysis of the experimental data in this section, and we begin by listing these predictions. Firstly, welfare should be improved through two channels: (i) a reduction in miscoordination; and (ii) lower thresholds used to switch from the not-attacking to attacking actions. The qualitative predictions are: (iii) if individuals' messages about intentions agree, their actions should coincide with their messages; and (iv) if individuals' messages disagree, they should employ a more demanding cutoff. Finally, the quantitative thresholds are calculated for the communication and action stages that can then be compared to estimates from the experimental data.[19]

Table 3: Average Payoffs

Treatments	T_0	T_S	T_I	$T_{I\&S}$	T_{DCT}
T_0	69.91	~ 70.00	~ 70.75	~ 69.31	$<^{**} 71.07$
		(-0.114)	(-2.298)	(0.465)	(-3.030)

Note: $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$. Welch t-statistic in parentheses.

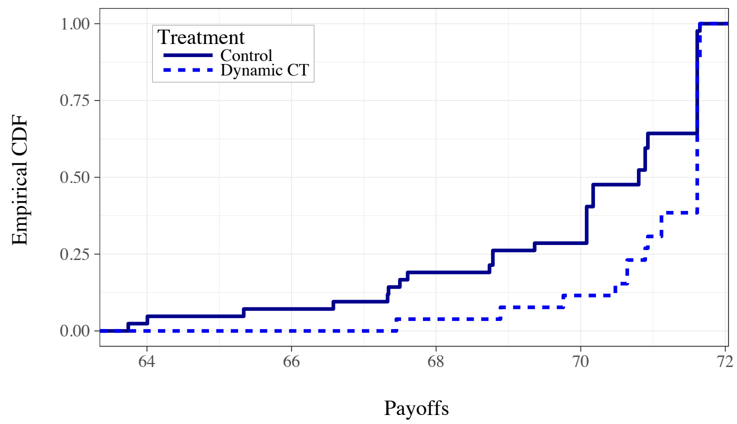
We evaluate the experimental effects of communication treatments on welfare by looking at aggregate average payoffs for each of the treatments. Table 3 presents the mean payoffs in experimental currency units for each treatment and the results of binary hypothesis testing of the control treatment versus all other communication treatments (p-values are adjusted using Bonferroni correction for multiple hypotheses testing).

Interestingly, one-stage communication treatments, T_S,T_I and $T_{I\&S}$, have no significant effect on average payoffs. Dynamic-cheap-talk treatment is the only communication protocol

[19] In the experiment, we used neutral language and we denoted attack and not-attack actions by alternatives A and B, respectively. For the purposes of consistency with the previous sections, I will continue to use A and N in the experimental results, even though the subjects' answers were A and B.

that provides significantly higher average payoffs compared to the control treatment. In addition to the statistically significant difference in means, the empirical CDF of payoffs in T_{DCT} first-order stochastically dominates the payoffs in T_0 (see Figure 5, where the Kolmogorov-Smirnov test rejects the hypothesis of equal distributions with $p < 0.05$).

Figure 5: Empirical CDFs of T_0 and T_{DCT}

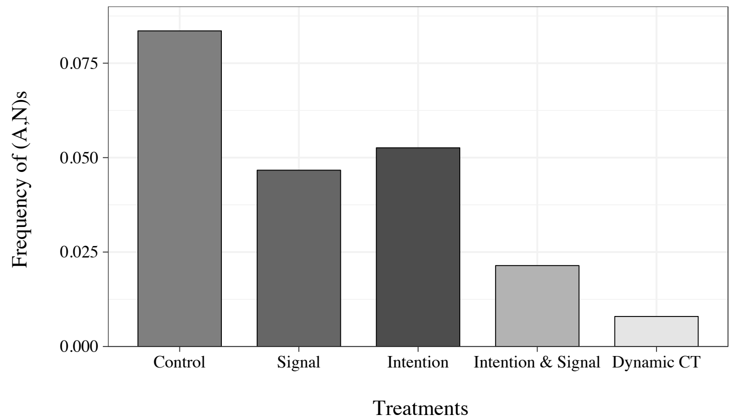


Allowing subjects to send cheap-talk messages corresponding to the actions has been shown to be effective in coordination games with complete information (see, e.g., Blume and Ortmann (2007)). However, no statistically significant differences exist for a similar one stage communication treatment in the coordination game with incomplete information studied in this paper. To further analyze this converse result, let us break down the effect of communication and look into two forces that drive the welfare improvement theoretically.

Consider first cases of miscoordination—subjects choosing different actions. For all the treatments, Figure 6 presents the frequency of mismatched actions. All the communication treatments provide less miscoordination compared to the control treatment, the most effective being the T_{DCT} treatment. In the dynamic-cheap-talk treatment, the communication and action stages are merged; this continuity between stages, and the ability to instantaneously adjust the intended action, leads to lowest level of miscoordination and the highest welfare improvement.

Despite the decrease in miscoordination, one-stage communication protocols have insignificant effects on average payoffs. We proceed by examining the movement from messages to actions to find out if this difference in the effects of one- and multi-stage treatments is due to qualitative features. Figure 7, provides a transition of all possible message pairs to actions for the treatment T_I, letter part of the treatment $T_{I\&S}$ and the initial choice of the treatment T_{DCT}. Recall that, according to the informative equilibrium, if messages agree, the follow up actions should be the intended actions. The experimental results strongly support this theoretical prediction. As we can see in Figure 7, if both subjects send a message A or both send a message N, the outcome is (A, A) or (N, N), respectively, more than 98% of the

Figure 6: Frequency of Miscoordination



time. This result demonstrates that subjects' message to action behavior is highly consistent with the theoretical predictions.

Theoretically, disagreement in messages should lead to either switching to the not-attacking action or following through on their communicated intentions if the positive signal is strong enough. Based on the experimental parameters, we should see zero-values in the elements of transition matrix when (A, N) turns into anything except (N, N).[20] Disagreements in messages result in all possible outcomes, but the largest mass, over 49%, is on (N, N) action pair. And similar to the agreement cases, all three treatments favor the theoretical results in the same way.

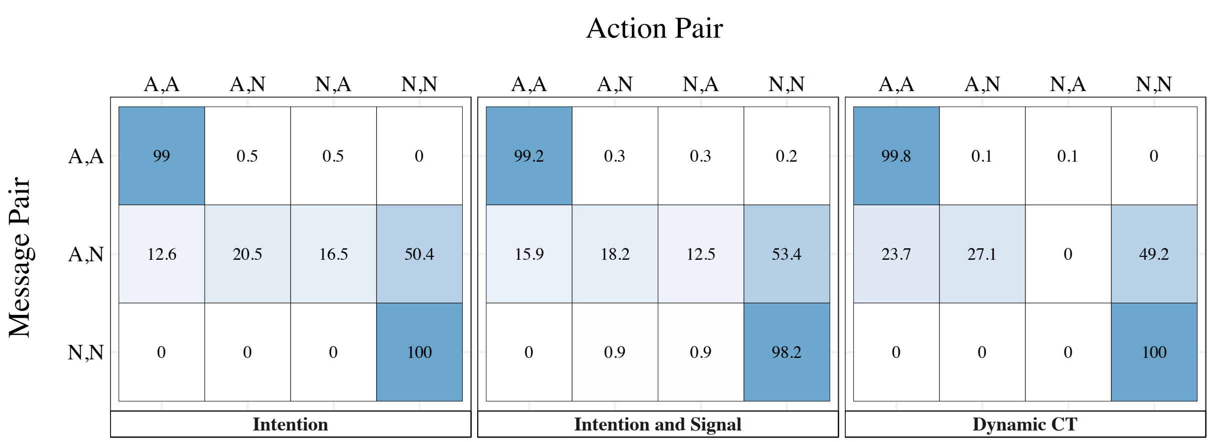


Figure 7: Transition matrices for treatments T_I, $T_{I\&S}$ and T_{DCT}

The results of T_{DCT} and treatments T_I and $T_{I\&S}$ are very similar, the only difference being in disagreeing messages. In the rightmost table of Figure 7, the third column, second row

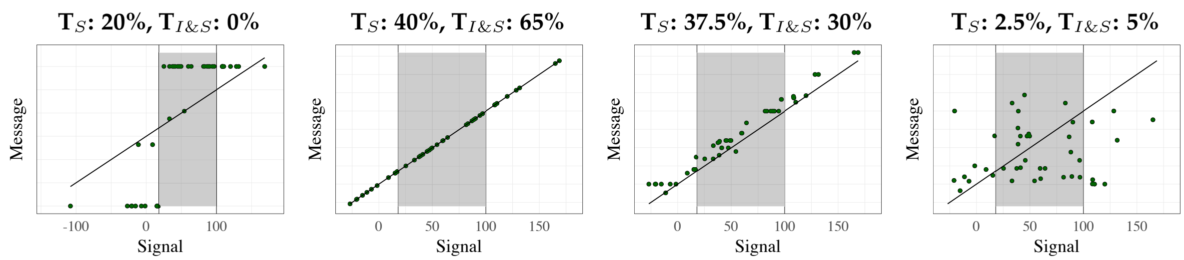
[20] If one player sends a message not to attack, then the other player should attack if their private signal is greater than 178.24. Since, the realized private signals and corresponding messages never appeared in that range then, theoretically, the second line of the transition matrix should be 0%, 0%, 0% and 100%.

reads 0%; hence, no (A, N) messages turned into (N, A) actions, compared to 16.5% and 12.5% in treatments T_I and $\mathrm{T}_{I\&S}$, respectively. The message pair (A, N) never translates into (N, A) and some of the mass is shifted to action pairs (A, A) and (A, N). This result suggests that giving subjects the ability to adjust messages continuously provides an opportunity to easily turn (A, N) to coordination (A, A).

The ability to instantaneously adjust messages reduces the strategic uncertainty caused by mismatched messages; subjects are, therefore, less likely to overweight the value of a message and disregard their own information. Overall, Figure 7 provides strong support for the qualitative predictions of the informative equilibrium across all relevant treatments. Therefore, message-to-action transitions are not the source of discrepancy in average payoffs between one- and multi-stage communication protocols.

7.1 Numeric Messages

Before analyzing the quantitative thresholds for all treatments, we examime numeric messages and classify them into four types: partition, truth-telling, mixed, and babbling. Figure 8 depicts the different strategy types on a graph where the x-axis is received signals, the y-axis is sent messages, the gray area depicts the coordination region, and the black line is the 45-degree line. The top part of Figure 8 presents the distribution of the types in two treatments: T_S, in which subjects' messages are numbers; and $\mathrm{T}_{I\&S}$, in which subjects' messages are numbers and actions.



(a) Partition (b) Truth-telling (c) Mixed (d) Babbling

Figure 8: Sample Message Strategies

If a subject partitions their signals into two messages for most of the 50 rounds, we call this subject a *partition* type. For example, Figure 8a presents the behavior of a subject classified as a partition type; this subject used the number 150 to indicate high signals and -150 to indicate low signals. In the number-message treatment, T_S, 20% of subjects find a common language to signal intentions to the other subject (some subjects use 1 and 2, others employ large and small numbers, 150 and -150, to indicate their intention for alternatives A and N, respectively). As the $\mathrm{T}_{I\&S}$ treatment subjects are allowed to send numbers accompanied with the letter A and N, there is no need to construct a new common language using numbers; therefore, there are no partition types in this treatment.

If a subject sends a message within five points of the true value in 45 out of 50 rounds, we classify this behavior as *truth-telling* and label the subject a *truth-telling* type. This is shown in Figure 8b. Consistent with the literature on information transmission in cheap-talk games, a fraction of subjects truthfully report their private information.[21] However, in treatments T_S and $T_{I\&S}$, 60% and 35% (resp.) of subjects employ strategies different from revealing the full information.

In Figure 8c, we see a case of partial truth-telling or, as we classify them, *mixed types*. These types tell the truth for some values of the realized signal; however, in other regions, they either partition or babble.[22] Finally, we have babbling types that send messages that seem to be unrelated to underlying signals, see Figure 8d. In treatments T_S and $T_{I\&S}$, respectively, there are only 2.5% and 5% of subjects whose behavior can be classified as babbling, providing strong support for informative equilibrium.

To calculate the thresholds using the experimental data, we need some preliminary results. In the next subsection we provide all required definitions and tools.

7.2 Quantitative Thresholds

We say that behavior is consistent with a threshold strategy if subjects use a perfect or almost perfect threshold strategy. An example of a *perfect threshold strategy* usage is presented in Figure 9a, in which alternative N is chosen for low realizations of signals and alternative A is chosen for high realizations of signals with *exactly one* switching point. For signals less than 40, the subject has chosen an action N ($a_i = 0$ on the graph), whereas for signals above 40 the choice is A ($a_i = 1$ on the graph). A subject uses an *almost-perfect threshold strategy* if the threshold rule admits a few errors—more precisely, if the overlap is less than three signals.[23] For example, Figure 9b provides an example of almost perfect threshold strategy with the overlap of two signal realizations.

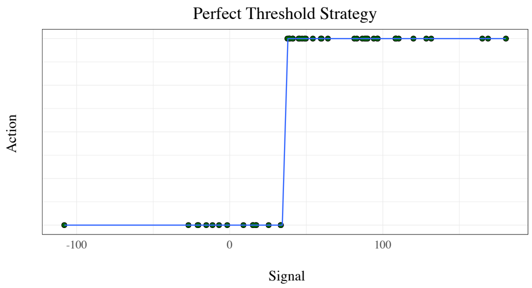
Result 1 summarizes the data. (For the breakdown of this result by each treatment and different periods, see Table 5 in Appendix B.) The data provide strong evidence of threshold strategy usage in both stages: while sending a binary message and, also, during the action stage.[24] Note that threshold strategies are robust in that even if subject believes other subject is using threshold strategy or randomizing, then the best response is still a threshold strategy.

[21] See, for example, Cai and Wang (2006), where the authors implement a cheap talk game from Crawford and Sobel (1982) in the lab. Similar results on truth-telling have been observed in other studies: Blume et al. (1998, 2001), Evans III et al. (2001), Gneezy (2005), Sánchez-Pagés and Vorsatz (2007). For more examples see Zak (2008). There are few studies that provide evidence of strategic information concealment, for example, Agranov and Schotter (2013, 2012) show that a vast majority of subjects refrains from truth-telling, especially in a disagreement region, where leader and followers face potential conflicts of interest. In general, the authors find that vague or ambiguous language improves coordination in a region where preferences are misaligned.

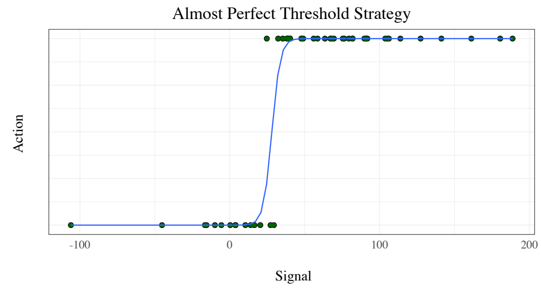
[22] The mixed types in this paper are similar to A and C types in Agranov and Schotter (2013).

[23] The classification is similar to the one given in Szkup and Trevino (2012)

[24] This result is consistent with previous literature, see Heinemann et al. (2004), Cornand and Heinemann (2014) and Szkup and Trevino (2012).



(a) Perfect Threshold Strategy



(b) Almost Perfect Threshold Strategy

Result 1 *98.28% of the subjects use threshold strategies in the binary-message stage, and 99.01% of the subjects use threshold strategies in the action stage.*[25]

To estimate the thresholds for all subject who use threshold strategies, a logistic distribution is fitted to the data of each subject and then averaged to provide aggregate results. The threshold is interpreted as a signal for which there is 50% chance of choosing either alternative. Recall that the CDF of the logistic distribution is given by

$$\Pr(A) = \frac{1}{1 + \exp(a - bx_i)}$$

with parameters $a \in \mathbb{R}$ and $b > 0$.

Following Heinemann et al. (2004), the ratio a/b can be interpreted as the mean threshold of the group. The standard deviation of the threshold estimate, $\pi/(b\sqrt{3})$, can be interpreted as a measure of coordination. Table 4 provides experimental and theoretical thresholds for sending binary messages and estimated thresholds for taking an attack action. For the action stage, the table provides unconditional thresholds (thresholds calculated using all of the data) and conditional thresholds (where thresholds are conditional on matching messages). In the rest of this section, using the information in the table 4, we provide results that shed light on the quantitative differences between theoretical and experimental thresholds.

Recall the average welfare effect of communication: the one-shot communication treatments provide no significant increase in average payoffs. Figure 10 illustrates the action thresholds on one line for all treatments. Theoretical thresholds are depicted by full bars and average experiment thresholds by dashed lines.[26] Now we can identify where the experimental data departs from theory. The welfare increase from threshold reduction is not attained by subjects in one-shot treatments; the subjects set much more demanding cutoff

[25] The calculations for binary messages are based on treatment T$_I$, the letter part of treatment T$_{I\&S}$, and the initial choice of treatment T$_{DCT}$. Calculations for the action stage include all treatments. The analyses uses the last 25 rounds.

[26] Under the parameters of the experiment, the theoretical threshold for sending a binary message is $x_C^* = 11.47$, and the theoretical thresholds in the action stage are $x^* = 11.47$ and $\bar{x}^* = 178.24$. We hypothesize that experimental thresholds will be similar to the theoretical predictions.

Table 4: Estimated and Theoretical Thresholds

Treatments	*Communication Stage*		*Action Stage*		
	Experimental	Theoretical	Unconditional	Conditional	Theoretical
T_0	—	—	26.84	—	28.31
			(1.653)		
T_S	—	—	29.06	—	11.47
			(4.196)		
T_I	25.62	11.47	28.66	25.80	11.47
	(3.214)		(3.855)	(0.752)	
$T_{I\&S}$	24.56	11.47	27.08	25.76	11.47
	(2.483)		(1.468)	(0.729)	
T_{DCT}	23.52	11.47	24.70	24.68	11.47
	(4.245)		(4.090)	(1.653)	

levels than the theory predicts. Moreover, the estimated thresholds with one-stage communication are clustered around the control threshold and they are not significantly lower the control treatment.

Theoretically, threshold reduction is achieved by considering the communication stage strategically. If an individual has information under which they would choose to attack if the other person were to attack, they will send a message that they are going to attack. This reasoning pushes down the threshold for sending the attack message. Notice that this welfare improving outcome is due to individuals' strategic behavior in the communication stage, stating that they are going to attack even when they are unsure of whether they will follow through. In the experiment, subjects instead seem to follow a simple heuristic when sending a message. The thresholds they use to send a message are similar to the one they would use in the action stage in the absence of communication. Hence, they state what they would have done in the absence of communication, and they miss out on the welfare improvement from threshold reduction.

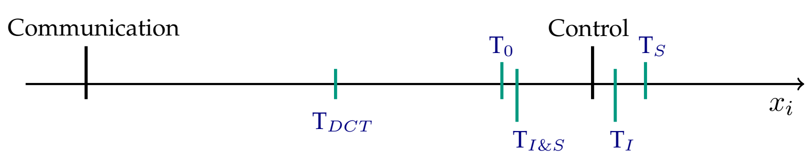


Figure 10: Threshold Comparisons

Result 2 *The threshold used to send binary messages, and the threshold used to take the attack action conditional on the other's message being attack, are statistically indistinguishable.*

We end this section with two further results on the quantitative thresholds. Theoretically, the threshold used to send a binary message should be identical to the threshold used to attack in the action stage given that the other player is attacking. Thus, the next conjecture concerns how consistent subjects are with setting their message and action thresholds, and whether they use the same cutoff for both decisions. No evidence is found to reject the hypothesis that the experimental estimates in the communication stage are statistically equivalent to conditional thresholds in the action stage (a Wilcoxon signed-rank test, which is a pairwise nonparametric test, cannot reject the hypothesis at the 5% significance level); hence, the next result.

Result 3 *The action thresholds used are statistically indistinguishable in treatments with intention sharing and with intention and signal sharing.*

For one-stage communication treatments, the theoretical thresholds for sending a message and then actions based on the messages are the same. However, these communication treatments might not be taken the same way behaviorally. Given the estimates in Table 4, we can test whether the communication treatments provide the same quantitative thresholds. The unconditional action-stage thresholds in treatments T_S, T_I, and $T_{I\&S}$ are statistically different with $p < 0.05$ (Kruskal-Wallis rank sum test, $\chi^2 = 6.6$). However, the conditional thresholds in the action stage and the binary-message thresholds in the communication stage are statistically indistinguishable in treatments T_I and $T_{I\&S}$.

8 Conclusion

Communication is a natural aspect of environments modeled by global games, and taking communication effects into account is important. In these environments, theoretically, communication can reduces global games' inherent inefficiency region and decrease miscoordination. The welfare gain is based on strategic behavior in the communication stage, and individuals need to understand they are better off by being strategic.

The experimental data supports qualitative features of the equilibrium. In the three treatments where subjects can use letters corresponding to two alternatives, if both subjects agree on an intended action, they follow through with their initial intentions in over 99% of the cases. This result is consistent with the theoretical prediction of using the same threshold for sending a message and then following through if the other individual agrees. However, the subjects set much more demanding cutoff levels than the theory predicts. The thresholds they use to send a message are similar to the one they would use in the action stage in the absence of communication. Hence, they state what they would have done in the absence of communication, and they miss out on the welfare improvement through reduction of the threshold.

Experimental results provide evidence that miscoordination is reduced; however, subjects miss out on significant payoff improvement through reduction of the thresholds. Although all communication treatments reduce miscoordination observed in the control treatment, the dynamic-cheap-talk treatment is the most effective communication protocol. This continuous interaction provides a significantly higher payoff compared to the baseline case.

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Appendices

A Proofs and Details

A.1 Expected Payoff

Recall the payoff structure:

$$g(\theta, x_j) := \begin{cases} \theta & \text{if } A(\theta, x_j) \text{ or } B(\theta) \\ 0 & \text{else} \end{cases}$$

where

$$A(\theta, x_j) := \{x_j \geq x_j^*\} \text{ and } \{\theta \geq \underline{\theta}\}$$
$$B(\theta) := \{\theta \geq \overline{\theta}\} \text{ and } \{\theta \geq \underline{\theta}\}$$

Condition $B(\theta)$ reduces to $\{\theta \geq \overline{\theta}\}$. Hence, we have

$$\mathbb{E}[g(\theta, x_j)|x_i] = \int g(\theta, x_j) p(\theta, x_j|x_i) d(\theta, x_j) = \int_{A \cup B} \theta p(\theta, x_j|x_i) d(\theta, x_j)$$

Using basic properties of conditional probability and addition rule of probability we get

$$\mathbb{E}[g(\theta, x_j)|x_i] = \int_{\underline{\theta}}^{\overline{\theta}} \theta \left[\Pr(x_j \geq x_j^*|x_i, \theta)\right] p(\theta|x_i) d\theta + \int_{\overline{\theta}}^{+\infty} \theta p(\theta|x_i) d\theta$$

A.2 Supermodular Game

In this section I show that the action stage game belongs to the class of monotone supermodular games of Van Zandt and Vives (2007) (parts of this section are very similar to Szkup and Trevino (2012), the main difference and complications come from the ESN posterior distribution in this paper compared to the normal distribution in Szkup and Trevino (2012)).

Let us set up the environment consistent with Van Zandt and Vives (2007). The set of players is $I = \{1, 2\}$, indexed by i. The type space of player $i \in I$ is a measurable space $(\Theta_i, \mathcal{F}_i)$. Te residual uncertainty not observed by the players is the state space $(\Theta_0, \mathcal{F}_0)$. Let $\mathcal{F}$ be the overall product sigma-algebra, let $\mathcal{F}_{-i}$ be the product sigma-algebra $\otimes_{k \neq i} \mathcal{F}_k$, $T := \Theta_0 \times \Theta_1 \times \Theta_2$ and $\Theta_{-i} := \prod_{k \neq i} \Theta_k$. For any player i, the interim beliefs are given by $p_i : \Theta \to \mathcal{M}_{-i}$, where $\mathcal{M}_{-i}$ is the set of probability measures on $(\Theta_{-i}, \mathcal{F}_{-i})$. The action set of player i is A_i and the action profiles $A = A_1 \times A_2$, where $A_{-i} = \prod_{i \neq j} A_j$. And finally, the utility function is $u_i : A \times \Theta \to \mathbb{R}$. In addition, type and action sets are nonempty.

Let us define a strategy for player i, $\varsigma_i : \Theta_i \to A_i$ is a measurable function, the set of strategies is Σ_i and the set of strategy profiles $\Sigma = \Sigma_1 \times \Sigma_2$. Now, we can define a Bayesian Nash equilibrium. A Bayes-Nash equilibrium is a strategy profile $\sigma \in \Sigma$ such that each player and each type chooses a best response to the strategy profile of the other players. Let P_i be the interim belief of type t_i, and the expected payoff of action a_i is

$$V_i(\sigma_i, t_i, P_{-i}; \sigma_{-i}) = \int\limits_{T_{-i}} u_i(a_i, \sigma_{-i}(t_{-i}, t_i, t_{-i}))dP_{-i}(t_{-i}).$$

We say that $\sigma \in \Sigma$ is a Bayesian Nash equilibrium if and only if, for $i \in I$ and $t_i\Theta_i$, $\sigma_i(t_i) \in \psi_i(t_i, p_i(t_i); \sigma_{-i})$, where

$$\psi_i(t_i, P_i(t_i); \sigma_{-i}) = \arg\max_{a_i \in A_i} V_i(a_i, t_i, P_{-i}; \sigma_{-i})$$

Finally, let us define the best response correspondence $\beta_i : \Sigma_{-i} \to \Sigma_i$:

$$b_i(\sigma_{-i}) = \{\sigma \in \Sigma_i | \sigma_i(t_i) \in \psi_i(t_i, p_i(t_i); \sigma_{-i}) \forall t_i \in \Theta_i\}$$

The strategy profile $\sigma_i \in b_i(\sigma_{-i}), i \in I$ is a Bayes-Nash equilibrium. There is additional structure on actions and types.

[i] For each player i, A_i is a complete lattice.

[ii] Type space Θ_k, $k = 0, 1, 2$ is endowed with partial order.

A strategy $\sigma_i \in \Sigma_i$ is monotone if $\forall t_i, t_i'$, such that $t_i \geq t_i'$, then $\sigma_i(t_i) \geq \sigma_i(t_i')$.

Assumption 1

[i] The function $t_i \to p_i(F_{-i}|t_i)$ is measurable for $i \in I$ and $F_{-i} \in \mathcal{F}_{-i}$

[ii] A_i is a compact metric space for $i \in I$

[iii] The utility function satisfies the following three conditions for $i \in I$:

- *$u_i(a, \cdot) : \Theta \to \mathbb{R}$ is measurable for all $a \in A$;*
- *$u_i(\cdot, t) : \Theta \to \mathbb{R}$ is continuous for all $t \in \Theta$;*
- *$u_i(\cdot, \cdot)$ is bounded.*

Assumption 2

[i] The utility function u_i is supermodular in a_i, has increasing differences in (a_i, a_{-i}), and has increasing differences in (a_i, t);

[ii] The mapping $p_i : \Theta_i \to \mathcal{M}_{-i}$ is increasing with respect to the partial order on $\mathcal{M}_{-i}$ of first order stochastic dominance.

Result 4 *(Van Zandt and Vives (2007)) Under the provided structure and given the assumptions (1) and (2), there exist a greatest and a least Bayesian Nash equilibrium, and each one is in monotone strategies.*

A.3 Proof of Theorem 1

A.3.1 Proof of Lemma 1

Proof. Recall that $m_i : X_i \to M_i$, $a_i : X_i \times M \to A_i$ and $u_i : A \times \Theta \to \mathbb{R}$, where $M = M_i \times M_j$, $A = A_i \times A_j$, for $i \in I$ and $i \neq j$. The expected utility can be written as

$$\int\limits_{\theta\in\Theta} a_i(x_i;(m_i(x_i),m_{-i}))\left[\theta\left(\mathbb{1}_{\{\theta\in[\underline{\theta},\bar{\theta})\}}a_j(x_j;(m_i(x_i),m_{-i}))+\mathbb{1}_{\{\theta\geq\bar{\theta}\}}\right)-c\right]p(\theta|x_i,(m_i(x_i),m_{-i}))d\theta \tag{9}$$

Let $\varsigma = (m, a, p)$ be a symmetric pure strategy perfect Bayesian equilibrium. Take x_1 and $x_2 \in X_i$, such that $x_1 < x_2$ and $m_i(x_1) \neq m_i(x_2)$ and let

$$\int\limits_{\theta\in\Theta} u(a(x_2;(m(x_2),m_{-i})),a_{-i};\theta)p(\theta|x_2,(m(x_2),m_{-i}))d\theta \geq \tag{10}$$
$$\int\limits_{\theta\in\Theta} u(a(x_1;(m(x_1),m_{-i})),a_{-i};\theta)p(\theta|x_1,(m(x_1),m_{-i}))d\theta$$

(The above conditions exclude the equilibria in which $m_i(x_i) = m_i(x_j)$ for all $x_i, x_j \in X_i$. Since, we are looking for an informative communication strategy, where some information is transmitted, the condition is without loss of generality.) Consider $x_3 \in X_i$, such that $x_3 > x_2$. Note, communication strategy m_i enters expected payoff function through $Pr(a_j = 1|\cdot,(m(\cdot),\cdot))$ and $p(\theta|\cdot,(m(\cdot),\cdot))$. Then, since $p(\theta|x_2,\mathcal{I}) > p(\theta|x_1,\mathcal{I})$, for any $\mathcal{I} \in M$ and given equation 10, we get

$$Pr(a_j = 1|x_2,(m(x_2),m_{-i})) \geq Pr(a_j = 1|x_1,(m(x_1),m_{-i})) \tag{11}$$

Then, equation 11 yields

$$\int\limits_{\theta\in\Theta} u(a(x_3;(m(x_2),m_{-i})),a_{-i};\theta)p(\theta|x_3,(m(x_2),m_{-i}))d\theta \geq \tag{12}$$
$$\int\limits_{\theta\in\Theta} u(a(x_3;(m(x_1),m_{-i})),a_{-i};\theta)p(\theta|x_3,(m(x_1),m_{-i}))d\theta \tag{13}$$

therefore, $m(x_3) = m(x_2)$. ■

For uniqueness result (in communication stage) we need a following assumption. If a player i is of type $x_i \in X_i$, such that in the action stage $a_i(x_i;\cdot,\mathcal{I}) = 0$ for all $\mathcal{I} \in M$, then player i's message is $m_i = 0$. This assumption is addressing the following issue. Consider some signal $x_N \in X_i$, for which player i will abstain from attacking irrespective of the received messages. Since messages are costless and the final action is $a_i = 0$, this player is indifferent between sending any message. Because of that, we can construct the following equilibrium. For all signals $x_i \in X_i \backslash \{x_N\}$, players follow the equilibrium described in the Theorem 1, but $x_i = x_N$ sends a message $m(x_N) = 1$. Since, $x_i = x_N$ is a measure zero event, it will not affect the best responses or the thresholds. We have constructed an informative equilibrium that is payoff equivalent to the equilibrium described in Theorem 1.

To deal with this issue, we can make an assumption stated above or we could introduce a small cost ε of sending a message $m_i \in M_i$. Let $\tilde{M}_i = M_i \cup \{\emptyset\}$ and make $m_i = \emptyset$ costless. Introduction of this cost does not influence the players' best responses, and the analysis is unchanged up to the slight change of thresholds. Instead of attacking cost of c, the analysis is as if the cost was $c + \varepsilon$.

A.3.2 Combining Binary and Continuous Signals

The state of the world θ is drawn from a normal distribution with mean θ_0 and variance σ_θ^2

$$\theta = \theta_0 + \varepsilon_\theta \sigma_\theta$$

player i's private signal is draw from a normal distribution with mean θ and variance σ_i^2

$$x_i = \theta + \varepsilon_i \sigma_i$$

The player j's signal is x_j and player i receives a message m_j

$$x_j = \theta + \varepsilon_j \sigma_j$$

$$m_j = \begin{cases} 1, & \text{if } x_j \geq x_C^* \\ 0, & \text{if } x_j < x_C^* \end{cases}$$

So the distribution of m_j is

$$m_j \sim \text{Bern}(1 - q(\theta))$$

where

$$q(\theta) = \frac{1}{\sqrt{2\pi}\sigma_j} \int_{-\infty}^{x_C^*} \exp\left(-\frac{(y-\theta)^2}{2\sigma_j^2}\right) dy$$

Lemma 3 *The density of m_j given θ can be written as*

$$p(m_j|\theta) = \Phi(\zeta_j \theta; \zeta_j x_C^*, \sigma_j^2)$$

where $\zeta_j := sgn(2m_j - 1)$.

Proof. As m_j is a Bernoulli-distributed random variable,

$$p(m_j|\theta) = (1 - q(\theta))^{m_j} \times q(\theta)^{1-m_j}$$

where $q(\theta; x_C^*, \sigma_j^2) := \Phi(x_C^*; \theta, \sigma_j^2)$. First, notice that $\Phi(x_C^*; \theta, \sigma_j^2) = 1 - \Phi(\theta; x_C^*, \sigma_j^2)$.

When $m_j = 1$, we have

$$p(m_j = 1|\theta) = \Phi(\theta; x_C^*, \sigma_j^2)$$

When $m_j = 0$:

$$p(m_j = 0|\theta) = \Phi(x_C^*; \theta, \sigma_j^2) = \Phi(-\theta; -x_C^*, \sigma_j^2)$$

Thus

$$p(m_j|\theta) = \Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j^2)$$

where $\zeta_j := \text{sgn}(2m_j - 1)$. ■

Lemma 4 *The likelihood function of θ, with data (x_i, m_j), is Extended Skew-Normal with parameters $ESN(X_i, \sigma_i, \alpha, \tau)$, and density*

$$p(\theta) = \frac{1}{\Phi(\tau)} \frac{1}{\sigma_i} \phi\left(\frac{\theta - x_i}{\sigma_i}\right) \Phi\left(\alpha_0 + \alpha \frac{\theta - x_i}{\sigma_i}\right)$$

where

$$\alpha := \zeta_j \times \sigma_i/\sigma_j, \quad \alpha_0 := \zeta_j \times (x_i - x_C^*)/\sigma_j, \quad \tau := \frac{\alpha_0}{\sqrt{1+\alpha^2}}$$

Proof. As x_i and m_j are conditionally independent, then, by Lemma 3,

$$p(x_i, m_j|\theta) = \phi(\theta; x_i, \sigma_i^2)\Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j^2)$$

As a function of θ,

$$p(\theta|x_i, m_j) \propto \phi(\theta; x_i, \sigma_i^2)\Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j^2)$$

Let $\tau := \frac{\alpha_0}{\sqrt{1+\alpha^2}}$. Then

$$\int \phi(\theta; x_i, \sigma_i^2)\Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j^2) d\theta = \Phi(\tau)$$

Thus

$$p(\theta) = \frac{1}{\Phi(\tau)} \frac{1}{\sigma_i} \phi\left(\frac{\theta - X_i}{\sigma_i}\right) \Phi\left(\alpha_0 + \alpha \frac{\theta - X_i}{\sigma_i}\right)$$

which is the pdf of an Extended Skew-Normal (ESN) distribution. ■

The likelihood is extended skewed normal with parameters $ESN(X_i, \sigma_i, \alpha, \tau)$, and the prior is $N(\theta_0, \sigma_\theta)$.

Proof of Lemma 2. Lemma 4 establishes the likelihood function of θ. With a normal prior for θ, we use the updating formulae in Azzalini (2013). ■

Mean and Variance

The moment generating function (eq 2.40, Azzalini (2013)):

$$\boxed{M(t) := \mathbb{E}\left\{\exp(\xi t + \sigma_i Z t)\right\} = \exp(\xi t + 0.5\sigma_i^2 t)\frac{\Phi(\tau + \delta\sigma_i t)}{\Phi(\tau)}}$$

The mean is $\mu = \frac{d}{dt}M(t)|_{t=0}$. Let's take the derivative

$$\frac{d}{dt}M(t) = \exp(\xi t + 0.5\sigma_i^2 t^2)\left[\xi + \sigma_i^2 t\right]\frac{\Phi(\tau + \delta\sigma_i t)}{\Phi(\tau)} + \exp(\xi t + 0.5\sigma_i^2 t^2)\frac{\phi(\tau + \delta\sigma_i t)}{\Phi(\tau)}(\delta\sigma_i)$$

Evaluate at $t = 0$,

$$\boxed{\mu = \frac{d}{dt}M(t)|_{t=0} = \xi + \frac{\phi(\tau)}{\Phi(\tau)}(\delta\sigma_i)}$$

When $\tau = 0$

$$\frac{d}{dt}M(t)|_{t=0} = \xi + \sqrt{\frac{2}{\pi}}(\delta\sigma_i)$$

Note that, in our case, we actually have

$$\int \theta\phi(\theta; X_i, \sigma_i)\Phi(\theta; x_C^*, \sigma_j)d\theta$$

which is missing the normalizing term $\Phi(\tau)$. So

$$\boxed{\int \theta\phi(\theta; X_i, \sigma_i)\Phi(\theta; x_C^*, \tau_j)d\theta = X_i\Phi(\tau) + \phi(\tau)\delta\sigma_i}$$

Now for the variance. Need the second derivative of $M(t)$:

$$\frac{d^2}{dt^2}M(t)|_{t=0} = \xi^2 + \sigma_i^2 + \xi\frac{\phi(\tau)}{\Phi(\tau)}(\delta\sigma_i) + \xi\frac{\phi(\tau)}{\Phi(\tau)}(\delta\sigma_i) - \frac{\phi(\tau)}{\Phi(\tau)}\tau(\delta\sigma_i)^2$$

Then

$$\sigma^2 = \frac{d^2}{dt^2}M(t)|_{t=0} - \left[\frac{d}{dt}M(t)|_{t=0}\right]^2 = \sigma_i^2 - \frac{\phi(\tau)}{\Phi(\tau)}\tau(\delta\sigma_i)^2 - \left[\frac{\phi(\tau)}{\Phi(\tau)}(\delta\sigma_i)\right]^2$$

or

$$\boxed{\sigma^2 = \sigma_i^2\left(1 - \frac{\phi(\tau)}{\Phi(\tau)}\delta^2\left[\tau + \frac{\phi(\tau)}{\Phi(\tau)}\right]\right)}$$

When $\tau = 0$:

$$\sigma^2 = \sigma_i^2\left(1 - \frac{1/(2\pi)}{0.5^2}\delta^2\right)$$

or

$$\sigma^2 = \sigma_i^2\left(1 - \frac{2}{\pi}\delta^2\right)$$

So we say that θ with pdf

$$p(\theta) = \frac{1}{\Phi(\tau)}\phi(\theta; X_i, \sigma_i)\Phi(\theta; x_C^*, \sigma_j)$$

is a random variable with an extended Skew-Normal distribution, and parameters

$$\alpha := \sigma_i/\sigma_j, \quad \delta := \alpha/\sqrt{1+\alpha^2}, \quad \alpha_0 := (X_i - x_C^*)/\sigma_j, \quad \tau = \frac{\alpha_0}{\sqrt{1+\alpha^2}}$$

which yields the standard notation of

$$p(\theta) = \frac{1}{\Phi(\tau)}\frac{1}{\omega}\phi\left(\frac{\theta-\xi}{\omega}\right)\Phi\left(\alpha_0 + \alpha\frac{\theta-\xi}{\omega}\right)$$

where $\xi := X_i$, $\omega := \sigma_i$.

The CDF. Using Eq. 2.49, Azzalini (2013):

$$\boxed{\Phi(x;\alpha,\tau) = \Phi(x) - \frac{1}{\Phi(\tau)}\left[H(x,\tau;\alpha) - H(\tau,x;\alpha)\right]}$$

where I've defined

$$H(y,z;\alpha) = T\left(y, \alpha + y^{-1}z\sqrt{1+\alpha^2}\right) - T\left(y, y^{-1}\tau\right)$$

and T is Owen's T-function:

$$T\left(h,a\right) = \frac{1}{2\pi}\int_0^a \frac{\exp(-0.5h^2(1+x^2))}{1+x^2}dx$$

An alternative representation using the bivariate normal distribution:

$$\boxed{\Phi(x;\alpha,\tau) = \frac{\Phi_B(x,\tau;-\delta)}{\Phi(\tau)}}$$

where

$$\Phi_B(x,y;\rho) = \int_{-\infty}^{x}\int_{-\infty}^{y}\phi(t)\phi\left(\frac{u+\delta t}{\sqrt{1-\delta^2}}\right)\frac{1}{\sqrt{1-\delta^2}}dudt$$

A.3.3 ML Estimator

If we did not have a closed form solution for the updating rule, we would have used the following MLE.

Log-likelihood is

$$\ln p(\theta|x_i, m_j) \propto \ln\phi(\theta; x_i, \sigma_i) + \ln\Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j)$$

Derivative of the log-likelihood:

$$\frac{d}{d\theta}\ln[p(\theta|x_i,m_j)] = (1/\sigma_i^2)(x_i-\theta)+\kappa'(\theta)$$

Set equal to zero and rearrange: the ML value of θ solves

$$\theta - \sigma_i^2\kappa'(\theta) = X_i$$

where we define

$$\kappa(\theta; x_C^*, \sigma_j^2) := \ln\left[\Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j)\right],$$

and so

$$\kappa'(\theta; x_C^*, \sigma_j^2) := \frac{\phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j)}{\Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j)}$$

Variance of $\hat{\theta}$

The second derivative of the log-likelihood w.r.t. θ is:

$$\frac{d^2}{d\theta^2}\ln[p(\theta|x_i,m_j)] = -1/\sigma_i^2+\kappa''(\theta)$$

where

$$\begin{aligned}\kappa''(\theta; x_C^*, \sigma_j^2) &= \frac{\phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j)}{\Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j)}\zeta_j(x_C^*-\theta)\sigma_j^2 + (-1)\left(\frac{\phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j)}{\Phi(\zeta_j\theta; \zeta_j x_C^*, \sigma_j)}\right)^2 \\ &= \kappa'(\theta; x_C^*, \sigma_j^2)[\zeta_j(x_C^*-\theta)\sigma_j^2 - \kappa'(\theta; x_C^*, \sigma_j^2)]\end{aligned}$$

The asymptotic variance is the inverse of Fisher's information matrix.

$$\begin{aligned}I(\theta) &= -\mathbb{E}_\theta\left(\frac{d^2}{d\theta^2}\ln[p(\theta|x_i,m_j)]\right) \\ &= -\mathbb{E}_\theta\left(-(1/\sigma_i^2)+\kappa''(\theta)\right) \\ &= 1/\sigma_i^2 - \kappa''(\theta)\end{aligned}$$

A.3.4 Results on Expected Payoff

Action stage expected payoff can be written as

$$V((\tilde{x}^*, x^*)|x_C^*, \mathcal{I}) = \int_{\underline{\theta}}^{\bar{\theta}} \theta Pr\left[x_j \geq x^*|\theta, x_C^*, \mathcal{I}\right] p(\theta|\tilde{x}^*, x_C^*, \mathcal{I})d\theta + \int_{\bar{\theta}}^{\infty} \theta p(\theta|\tilde{x}^*, x_C^*, \mathcal{I})d\theta - c$$

$\mathcal{I} = (m(\tilde{x}^*), m(x^*)) \in M.$

Posterior belief

$$p(\theta|\tilde{x}^*, x_C^*, \mathcal{I}) = \frac{p(\tilde{x}^*, x_C^*, \mathcal{I}|\theta)p(\theta)}{\int\limits_{\Theta} p(\tilde{x}^*, x_C^*, \mathcal{I}|\theta)p(\theta)d\theta},$$

$\tilde{x}^* = x_C^*$ solves $V((\tilde{x}^*, x_C^*)|\mathcal{I}_1) = c$, where

$$\begin{aligned} V((\tilde{x}^*, x_C^*)|\mathcal{I}_1) &= \int\limits_{\underline{\theta}}^{\bar{\theta}} \theta \underbrace{Pr\left[x_j \geq x_C^*|\theta, x_C^*, \mathcal{I}_1\right]}_{=1} p(\theta|\tilde{x}^*, x_C^*, \mathcal{I}_1)d\theta + \int\limits_{\bar{\theta}}^{\infty} \theta p(\theta|\tilde{x}^*, x_C^*, \mathcal{I}_1)d\theta - c \\ &= \int\limits_{\underline{\theta}}^{\infty} \theta p(\theta|\tilde{x}^*, \mathcal{I}_1)d\theta - c \end{aligned}$$

$\tilde{x}^* = \bar{x}^*$ solves $V((\tilde{x}^*, x_C^*)|\mathcal{I}_0) = c$, where

$$\begin{aligned} V((\tilde{x}^*, x_C^*)|\mathcal{I}_0) &= \int\limits_{\underline{\theta}}^{\bar{\theta}} \theta \underbrace{Pr\left[x_j \geq x_C^*|\theta, x_C^*, \mathcal{I}_0\right]}_{=0} p(\theta|\tilde{x}^*, x_C^*, \mathcal{I}_0)d\theta + \int\limits_{\bar{\theta}}^{\infty} \theta p(\theta|\tilde{x}^*, x_C^*, \mathcal{I}_0)d\theta \\ &= \int\limits_{\bar{\theta}}^{\infty} \theta p(\theta|\tilde{x}^*, x_C^*, \mathcal{I}_0)d\theta \end{aligned}$$

Consider the case when $\mathcal{I} = \mathcal{I}_1$ and symmetric action stage threshold is x^*

$$V((x^*, x^*)|x_C^*, \mathcal{I}_1) = \int\limits_{\underline{\theta}}^{\bar{\theta}} \theta Pr\left[x_j \geq x^*|\theta, x_C^*, \mathcal{I}_1\right] p(\theta|x^*, x_C^*, \mathcal{I}_1)d\theta + \int\limits_{\bar{\theta}}^{\infty} \theta p(\theta|x^*, x_C^*, \mathcal{I}_1)d\theta - c$$

If the expression $\frac{dV((x^*,x^*)|x_C^*,\mathcal{I}_1)}{dx^*}$ is always positive, then there is a unique value of x^* solving $V(x^*, x^*|x_C^*, \mathcal{I}_1) = 0$ and the unique strategy surviving iterated deletion of strictly dominated strategies is a threshold rule with a cutoff x^*. In addition, since we know that $V((x_C^*, x_C^*)|x_C^*, \mathcal{I}_1) = 0$, then we get the unique cutoff $x^* = x_C^*$.

Lemma 5 $\frac{dV((x^*,x^*)|x_C^*,\mathcal{I}_1)}{dx^*} > 0$.

Proof.

$$\begin{aligned} \frac{dV((x^*, x^*)|x_C^*, \mathcal{I}_1)}{dx^*} &= \int\limits_{\underline{\theta}}^{\bar{\theta}} \theta \left(Pr\left[x_j \geq x^*|\theta, x_C^*, \mathcal{I}_1\right] \frac{\partial p(\theta|x^*, x_C^*, \mathcal{I}_1)}{\partial x^*} + p(\theta|x^*, x_C^*, \mathcal{I}_1)\frac{\partial p(\theta|x^*, x_C^*, \mathcal{I}_1)}{\partial x^*} \right) d\theta \\ &+ \int\limits_{\bar{\theta}}^{\infty} \theta \frac{\partial p(\theta|x^*, x_C^*, \mathcal{I}_1)}{\partial x^*} d\theta \end{aligned}$$

$$\geq \int_{\bar{\theta}}^{\infty} \frac{\phi(\tau_c)}{\Phi(\tau_c)} \frac{\zeta}{\sqrt{1+\alpha_c^2}} \underbrace{\left(\frac{1}{\sigma_j(1+\sigma_\theta^2/\sigma_i^2)} - \frac{\sqrt{1+\alpha^2}}{\sigma_i^2+\sigma_j^2} \right)}_{>0, \text{ if } \gamma(\sigma_i,\sigma_\theta)>\sqrt{2}} d\theta$$

$$+ \int_{\bar{\theta}}^{\infty} \zeta \frac{\phi(\tau_c\sqrt{1+\alpha_c^2}+\alpha_c\frac{\theta-\xi_c}{\omega_c})}{\Phi(\tau_c\sqrt{1+\alpha_c^2}+\alpha_c\frac{\theta-\xi_c}{\omega_c})} \underbrace{\left(\frac{\sqrt{1+\alpha^2}}{\sigma_i^2+\sigma_j^2} - \frac{1}{\sigma_j(1+\sigma_\theta^2/\sigma_i^2)} \right)}_{>0, \text{ if } \gamma(\sigma_i,\sigma_\theta)>\sqrt{2}} d\theta$$

$$\left(\text{If } \gamma(\sigma_i,\sigma_\theta) := \frac{r(r^2+1)}{\sigma_\theta} > \sqrt{2}, \text{ where } r := \frac{\sigma_\theta}{\sigma_i}, \text{ then} \right)$$

$$> 0$$

■

Lemma 6 *Conditional on attacking in the action stage, restricted expected payoff function $V\left(x_i, x_j | x_C^*, \mathcal{I}\right)$ is increasing in x_i and it is decreasing in x_j.*

Proof. First, consider the case of $\mathcal{I} = \mathcal{I}_1$, then

$$V((x,x^*)|x_C^*, \mathcal{I}_1) = \int_{\underline{\theta}}^{\infty} \theta p(\theta|x, x_C^*, \mathcal{I}_1) d\theta - c$$

and

$$\frac{\partial V((x,x^*)|x_C^*, \mathcal{I}_1)}{\partial x} = \int_{\underline{\theta}}^{\infty} \theta \frac{\partial p(\theta|x, x_C^*, \mathcal{I}_1)}{\partial x} d\theta \geq 0$$

Similarly, if $\mathcal{I} = \mathcal{I}_0$, then

$$\frac{\partial V((x,x^*)|x_C^*, \mathcal{I}_0)}{\partial x} = \int_{\bar{\theta}}^{\infty} \theta \frac{\partial p(\theta|x, x_C^*, \mathcal{I}_0)}{\partial x} d\theta \geq 0$$

Finally, for any $\mathcal{I} \in M$

$$\frac{\partial V((x,x^*)|x_C^*, \mathcal{I})}{\partial x^*} = \int_{\bar{\theta}}^{\infty} \theta \frac{\partial p(\theta|x, x_C^*, \mathcal{I})}{\partial x^*} d\theta = 0 \leq 0$$

■

A.3.5 Arbitrary Message Space

Communication strategy is $m : \mathbb{R} \to M$, where M is arbitrary message space.

Lemma 7 *In an informative equilibrium, M contains two messages, m_N and m_A.*

Proof. Consider an informative BNE and let $X_1 = \{x \in \mathbb{R} | Pr(a_i(x) = 1|m) = 0, \forall m \in M\}$. Let $x^* \in \sup_x X_1$, $x^{'} > x^*$ and $m_N := m(x), x \in X_1$. Since the equilibrium is informative, $\exists m^{'} \in M$, such that $m^{'} \neq m_N$ and $Pr(a_j(x^{'}) = 1|m^{'}) > 0$. Let $m_A \in \text{argmax}_{m \in M} Pr(a_j(x^{'}) = 1|m)$. Suppose $\exists m^{'} \in M$, $m' \neq m_A$ and $Pr(a_j(x^{'}) = 1|m^{'}) > Pr(a_j(x^{'}) = 1|m_A)$. Then, $V_i(x^{'}, \cdot|\cdot, m^{'}) > V_i(x^{'}, \cdot|\cdot, m_A)$. This contradicts the assumption that $m(x^{'}) = m_A$. ∎

B Extra Figures and Tables

B.1 Figures

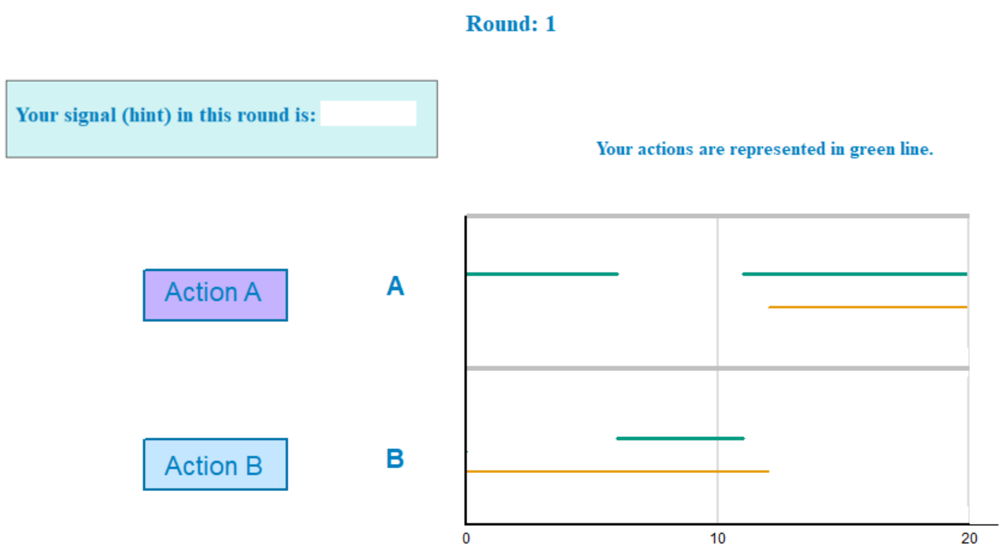

Figure 11: Sample Screen of Instant Revisions

B.2 Tables

Breakdown of threshold strategies by rounds and types.

Treatments	Rounds	Threshold Strategy	Perfect	Almost Perfect
T_I	All 50	98.0%	28.0%	70.0%
	Last 25	98.0%	88.0%	10.0%
$T_{I\&S}$	All 50	87.5%	32.5%	55.0%
	Last 25	97.5%	90.0%	7.5%
T_{DCT}	All 50	92.31%	30.77%	61.54%
	Last 25	100%	84.62%	15.38%

Table 5: Threshold strategy usage

B.3 Survey Results

Variable		%
Gender: Female		44.44
Game Theory: Yes		15.66
GPA (self reported)		3.5
Major:	Computer Science	17.68
	Economics	10.61
	Humanities	9.091
	Math	5.051
	Physics/Chemistry	2.525
	Other	54.55

Table 6: Survey Summary