Abstract

In this paper, we investigate market design for online gaming platforms. A significant fraction of such platforms’ revenue is generated by advertisements, in-app purchases, and subscriptions. Thus, it is necessary to understand which factors influence how much time users spend on the platform. We focus on one such factor - the outcome of the previous game. Using data from an online chess platform, we find strong evidence of history-dependent stopping behavior. We identify two primary types of players: those who are more likely to stop playing after a loss and those who are more likely to stop playing after a win. We propose a behavioral dynamic choice model in which the utility from playing another game is directly affected by the previous game’s outcome. We structurally estimate this time non-separable preference model and then conduct counterfactual analyses to evaluate alternative market designs. In the context of online chess games, a matching algorithm that incorporates stopping behavior can substantially alter the length of play.

**JEL Classification:** D9, D47, C5, C13;

**Keywords:** Online gaming platform design, time non-separable preferences, history dependent stopping behavior, chess.com.
1 Introduction

The online gaming industry grossed $162.3 billion in 2020 and is predicted to reach a gross annual revenue of $295.6 billion by 2026. The driving force of this growth is mobile gaming industry, which accounted for almost 50 percent of video gaming revenue worldwide in 2020. This paper documents a way to further game creators’ objectives by incorporating a market design that leverages information about users’ stopping behavior. In particular, we provide a way to encourage users on the platform to play more games. The ability to influence users’ stopping decisions affects the platform’s crucial objectives; we highlight two: ad revenue and user retention.

We collected data from the most prominent online chess platform, chess.com, which has over 50 million users; the site hosts, on average, 11 million chess games per day. We selected a random sample of users and scraped the entire history of their play in 2017 and 2018. Based on the 2017 data, about 79% of players fall into one of two behavioral types based on their stopping behavior. Among this group, about 30% are win-stoppers (players who are substantially more likely to stop playing after a win), and 70% are loss-stoppers (players who are substantially more likely to stop playing after a loss). Then we reclassify the same players using the 2018 data; we find that the vast majority of individuals are stable over time in terms of their stopping behavior. Therefore, platforms can leverage information on stopping behavior and utilize the following pattern: loss-stoppers play more when they win, while win-stoppers play more when they lose. Consequently, by increasing or decreasing the user’s chances of winning a game, the platform can alter the likelihood of the user playing another game.

We develop a theoretical framework to further study and quantify the impact of changing the likelihoods of winning for different types. Our structural model allows for time non-separable preferences, in which future game utility can depend on the history of play. The estimates from the structural model are consistent with the above-mentioned reduced-form evidence. For some people, a loss in a given game decreases the utility of playing another game, while for others, it increases the utility from playing another game. These distinct effects suggest the benefits of altering the likelihood of winning. We propose the platform modify the matching mechanism, which they fully control, based on heterogeneous types.

We use the structural model to conduct counterfactual analyses that assess the out-

\footnote{Source: www.mordorintelligence.com and www.statista.com.}
comes of using different matching algorithms. The platform currently prioritizes matching similarly rated players. We show that matching win-stoppers with, on average, more challenging opponents increases the average number of games played. For example, a pairing that decreases a win-stopper’s winning percentage from 50% to 45% (40%) increases the average number of games played in a session by 3.75% (6%). Similarly, a pairing that increases a loss-stopper’s winning percentage from 50% to 60% (65%) can increase the average number of games played by a loss-stopper during a session by 1% (7.5%). To put these numbers in perspective, over one year, a 5% increase in session length results in the average user playing 45 more games, which translates to 6 hours and 37 minutes more time spent on the platform.

The vast majority of all gaming platforms’ goal is to become and remain popular among users and gain profits from multiple sources, including in-app advertising, subscriptions, or sponsorship. So, we ask, how could the platform use the information about behavioral types of the user? First, more games user plays on a platform, the more chances the platform has to show ads. Second, a crucial aspect of the online chess experience is how long it takes to be matched with another player. A platform’s ability to find a match within a player’s rating range in a timely fashion is crucial for the user experience. The findings of this study can help platforms increase market thickness, which is especially important during times when there are not a sufficient number of users online.

This paper contributes to a growing body of literature on structural behavioral economics by structurally estimating a model with heterogeneous time non-separable preferences. Fundamentally, this paper presents and estimates a dynamic discrete choice model in which the agent may have non-separable preferences over the stochastic outcomes of their actions. In that sense, the application is analogous to the optimal stopping problems faced by, for example, taxi drivers, whose decisions to end their shifts may be influenced by their recent fares (see Camerer et al. (1997)). Recent empirical research on this topic is complicated by spatial search frictions and is limited by the imperfect observability of both decision-makers’ identities and histories of the outcome. In contrast, this analysis allows us to perfectly observe the actions, outcomes, and independent realizations of each agent’s decision problem. We take advantage of these rich data to demonstrate that an agent’s decisions cannot be reconciled in a model without non-separable preferences and that there is


See also Thakral and Tô (2017); Frechette et al. (2019); Farber (2005, 2008, 2015); Abeler et al. (2011); Morgul and Ozbay (2015); Cerulli-Harms et al. (2019)
substantial heterogeneity in preferences across players.

Researchers have used data from chess games to study risk, time, and other behavioral preferences for different age and gender groups. The closest parallel to the current study is a paper by Anderson and Green (2018) in which the authors use data on blitz games played on Free Internet Chess Server (FICS) between 2000 and 2015. The authors show that players are more likely to stop playing after they set a new personal best rating. This is an interesting result, however, players rarely set such records. Anderson and Green (2018) show that, on average, players reach a new personal best twice in 15 years. In contrast, the current study focuses on a phenomenon—the outcome of the previous game—that affects an agent’s decision after every game. This approach provides information that is more useful for the platform.

This paper is related to the literature on psychological biases. At least four identified biases—gambler’s fallacy, hot hand fallacy, source of motivation, and reference dependence—could be candidates to explain the behavior found in the data. Gambler’s fallacy and hot hand fallacy can each account for one of the stopping patterns but not the other. Rabin and Vayanos (2010) develop a model and show that an individual who exhibits gambler’s fallacy can also exhibit hot hand fallacy. While Rabin and Vayanos (2010) reconcile two seemingly opposite biases within an individual, this paper focuses on heterogeneity across individuals. Further, we do not make an argument about the underlying psychological forces that motivate the patterns observed in the data, but rather propose a theory that explains the decisions found in the data and allows for a unified framework that captures observed heterogeneity across players.

The analysis also contributes to the literature on the source of motivation, particularly the effects of wins and losses on future behavior. The existing findings in this literature are mixed. For example, Haenni (2019) and Cai et al. (2018) showed that past failure has a discouraging effect among amateur tennis players and workers, respectively. In contrast, in

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4 See the following recent papers that use chess data to study economic behavior: Gerdes and Gränsmark (2010), Gränsmark (2012), Dreber et al. (2013a,b), Bertoni et al. (2015), Linnemer and Visser (2016).
5 The blitz games analyzed in Anderson and Green (2018) lasted between 6 to 30 minutes. On chess.com, blitz is a type of chess game in which each player has a specific amount of time (between 3 and 10 minutes) for the entire game.
6 See Tversky and Kahneman (1991); Tversky and Gilovich (1989); Aharoni and Sarig (2012); Arkes (2010, 2013); Avugos et al. (2013); Cervone et al. (2014); Clotfelter and Cook (1993); Brown and Sauer (1993); Camerer (1989); Croson and Sundali (2005); Suetens et al. (2016); Gilovich et al. (1985); Green and Zwiebel (2017); Koehler and Conley (2003); Sundali and Croson (2006).
7 Miller and Sanjurjo (2018) found evidence that hot hand fallacy is not a fallacy in the context of basketball free throws.
a study of NBA and NCAA basketball players, Berger and Pope (2011) find an encouraging effect of being slightly behind at half time. We deviate from this literature in two ways: First, we examine the more immediate impact of past outcomes. Second, we focus on heterogeneity among players rather than an overall average effect. We find that losses have encouragement effects for some individuals and discouragement effects for others.

There is ample evidence from a variety of environments for reference dependence in decision making. For example, researchers have explored this phenomenon for cab drivers’ labor supply (Crawford and Meng (2011)), professional golf players’ effort choice (Pope and Schweitzer (2011)), risky choices in the “Deal or No Deal” game (Post et al. (2008)), domestic violence (Card and Dahl (2011)), and police performance after a lower than expected pay raise (Mas (2006)). We examined two types of reference dependence. First, we assume the reference point is a player’s rating at the start of a session. Second, we examine the case when the reference point is the expectation of winning based on the opponent’s rating. That is, if the opponent has a higher rating, then the player is more likely to have an expectation of losing and vice versa. We find that even though both reference points have statistically significant effects, the magnitudes of these effects are fairly limited—roughly 17 to 70 times smaller than the impact of the last game outcome depending on the behavioral type and the type of reference point.

The rest of the paper is organized as follows: Section 2 provides details on the data collection process and presents descriptive results. In Section 3, we introduce the structural model and the identification strategy. In Section 4, we show the results of the structural estimation and counterfactual analysis. In Section 5, we assess the time stability of behavioral types and the validity of our structural modeling choices using the Cox proportional hazards model. Section 6 concludes the analysis. We provide further robustness checks in the appendix.

2 Data and Descriptive Results

In this section we provide information about the chess.com platform and outline the data collection process. We then present descriptive statistics, highlighting patterns that suggest history dependence and heterogeneity in stopping behavior.

2.1 Data Collection

We scraped the data from chess.com, an online chess platform that is the most frequently visited chess website in the world. The website, which has over 50 million users, hosts
around 11 million chess games every day. Users range from amateur players to the world’s best chess players, including Magnus Carlsen, the World Chess Champion (as of 2021). The platform is free and anyone can register to play against either a human or computerized opponent using the website or a phone app. In addition to playing matches, users can take lessons and solve chess puzzles on the platform.

We collected the data in two steps. First, we gathered usernames from the platform without placing any restrictions on the history of play for that username. Using the selenium package in Python and the chess.com Application Programming Interface (API), we gathered a total of 1,793,473 usernames. Second, we collected users’ game histories against human players. We used chess.com API to download data for a sub-sample of users. We limited the analysis to a sub-sample of users because downloading data for all users would likely have resulted in being blocked by the web page. We randomly selected 1000 usernames at a time and scraped their history of play for 2017. We repeated this procedure 41 times. Next, we collected the history of play for the same users from 2018.

Each observation in the data set includes the following information about the user and the game characteristics: username, self-identified country of association, user’s platform rating, the length of the game, the type of game played, which user had white pieces, the start and end times of the game, and the outcome. Table 1 presents a summary of the data.

As part of the data cleaning process, we remove users who had not played any games in 2017. Because our focus is on relatively fast decisions, we remove daily games, which are unusually long, from the sample for the analysis in Section 2.3. No other restrictions are placed on the data for the analysis in Section 2.3. In cases where we impose additional restrictions during analysis, we include the pertinent details in the respective section.

2.2 Definitions

A *game* $g$ is a single game played against a human opponent. A collection of games ordered by time stamp, $(g_1, g_2, \ldots, g_n)$, is called a *session* if no game was played $T$ minutes before

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8 Because we did not place any restrictions at this stage, some of the usernames had not played any games in 2017, the focal year. Some usernames were created and never used or used only in years beyond the study period.

9 At a certain point, we stopped the data collection because we could no longer reach the website history.

10 In Section 2.2 we provide a formal definition of a session; alternative definitions and results are in Appendix B.1.

11 The average number of sessions was calculated in two steps. First, we calculated the number of sessions for each user in the data. Second, we averaged across all users. Similarly, the average session length and the average rating was calculated in two steps.
Table 1: Data description

<table>
<thead>
<tr>
<th>Games</th>
<th>50,165,970</th>
<th>Average Number of Sessions</th>
<th>630</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sessions</td>
<td>13,237,558</td>
<td>Average Session Length</td>
<td>5.11</td>
</tr>
<tr>
<td>Users</td>
<td>20,997</td>
<td>Average Rating</td>
<td>1.218</td>
</tr>
<tr>
<td>Rated games</td>
<td>99.7%</td>
<td>Pr(Win</td>
<td>White)</td>
</tr>
<tr>
<td>Game types:</td>
<td></td>
<td>Pr(Win</td>
<td>Black)</td>
</tr>
<tr>
<td>Blitz</td>
<td>71.9%</td>
<td>Pr(Win)</td>
<td>48.9</td>
</tr>
<tr>
<td>Bullet</td>
<td>21.7%</td>
<td>Pr(Loss)</td>
<td>47.9</td>
</tr>
<tr>
<td>Daily</td>
<td>2.2%</td>
<td>Pr(Draw)</td>
<td>3.2</td>
</tr>
</tbody>
</table>

$g_i$ or after $g_n$, and for any $i \in \{1, ..., n - 1\}$, the time between $g_i$ and $g_{i+1}$ is less than $T$.\footnote{For the main analysis we set $T = 30$ minutes; we then vary $T$ to check the robustness of the results and find no substantial differences. See Appendix B.1 for more details.} We call sessions that contain only one game ($n = 1$) only game (O-game). For sessions with $n \geq 2$, $g_1$ is the first game, $g_n$ is the last game and any game between the first and the last is referred to as a middle game. Based on the terms defined above, we categorize games into four mutually exclusive groups: only (O), first (F), middle (M) and last (L) games.

Let $f_W(\cdot)$ be a function that calculates the winning percentage in a particular type of game; for example, $f_W(L)$ is a user’s winning percentage in the last-games. In some cases, when the context is clear, instead of writing $f_W(F)$, $f_W(M)$, $f_W(L)$, $f_W(O)$, we write $F$, $M$, $L$, and $O$, to indicate the winning percentage in first, middle, last, and only-games, respectively.

2.3 Descriptive Results

In this section, we first establish that session-stopping behavior is history dependent. We then provide evidence of heterogeneity in stopping behavior and define behavioral types.

2.3.1 History Dependence

Our null hypotheses is that a user decides to stop the game randomly, in other words, stopping behavior is history independent:

$H_0$: Users’ stopping behavior is independent of the outcome of the last game.

In this case, the winning percentage in the last game should be similar to the winning percentage in any other type of game.

For each user, we calculated the winning percentages in the first, middle, and last-
games, as defined in Section 2.2. Figure 1 illustrates the relationship between the winning percentages in last-games and middle-games with each point depicting one user (the solid line represents the linear regression line).

$H_0$ implies that the correlation between the winning percentages in last-games and middle-games will be close to 1. The correlation between the winning percentages in last-games and middle-games is $-0.49$ and is statistically different from 1 with $p < 0.001$. Thus, at the aggregate level, the decision to stop is not random and we reject $H_0$.

Figure 1: Winning percentage by game category

2.3.2 Behavioral Types

We reject the null hypothesis of history independence; however, the alternative hypothesis does not specify the nature of the relationship between the outcome of the game just played and the decision to play another game. Further analysis is needed to determine whether users are more likely to end a session after a win or after a loss.

A closer inspection of Figure 1 reveals an intriguing pattern: some people have a much higher winning percentage in last-games than in middle-games while others have a much lower winning percentage in last-games than in middle-games. We introduce the following definition to elucidate this heterogeneity and classify users into mutually exclusive types:

**Definition 1** A user is a behavioral type at the tolerance level of $\tau$ if $f_W(L) > f_W(M) + \tau$ or $f_W(L) < f_W(M) - \tau$. She is:

• a win-stopper if $f_W(L) > f_W(M) + \tau$;
• a loss-stopper if $f_W(L) < f_W(M) - \tau$.

A user is a neutral type if $f_W(L) \in [f_W(M) - \tau, f_W(M) + \tau]$. 8
Using data from sessions that lasted two or more games, we classify users according to Definition 1 (see Figure 8 in the appendix for population decomposition by type using different tolerance levels of $\tau$). At a tolerance level, $\tau = 7\%$, we find that $79\%$ of users are behavioral types. Within this group, about $30\%$ are win-stoppers and $70\%$ are loss-stoppers.

2.3.3 Predictions

To further test the results above we take Definition 1 to the extreme and then we derive and test number of predictions. Let us assume that a win-stopper always stops after a win and a loss-stopper always stops after a loss. This extreme definition has number of implications, but first, let us focus on the following two (see Figure 2). First, the winning percentage for win-stoppers in both only and last-games must be 100. Second, the winning percentage for loss-stoppers in both only and last-games must be 0.

This is because if a win-stopper wins the initial game, she ends the session and the game is classified as the only game. If this user loses the first game, she will start another game, making this session at least two games long, so not an only game. Therefore, win stopper’s only-games are always wins. In addition, whenever the extreme win-stopper wins, she ends the session and we classify that game as the last game if the session is at least two games long. Therefore the winning percentage in the last game is 100. Following similar logic for loss stoppers we get that, the winning percentage for loss-stoppers in both only and last-games must be 0.

Combining these observations leads to the following prediction:

**Prediction 1** *The correlation between the winning percentages in last-games and only-games is positive.*

![Figure 2: Actions of extreme behavioral types](image-url)
Figure 3a presents a scatter plot of the winning percentages in last-games and only-games. A strong and significant positive relationship between the two winning percentages implies that individuals who are more likely to stop playing on a win (loss)—in other words, those who have a high winning (losing) percentage for last-games—also have a higher winning (losing) percentage in only-games, as stated in Prediction 1.

Furthermore, we find that the winning percentage for only-games is more than one and a half times higher for win-stoppers (64.0%) than for loss-stoppers (38.8%). Given that the average winning percentage in all games is 50.9% for win-stoppers and 50.4% for loss-stoppers, we can rule out the possibility that win-stoppers are simply better users and there is no link between their stopping behavior and the outcome of their last game.\textsuperscript{13}

Appendix D presents winning percentages and standard deviations for each behavioral type and game category.

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\textsuperscript{13} Appendix D presents winning percentages and standard deviations for each behavioral type and game category.
So far, we have used information on winning percentages for users’ middle, last, and only-games. We have not utilized information about their first-games. Figure 2 shows what each extreme type would do if they won or lost the initial game. This diagram illustrates the relationship between the winning percentages in first and last-games. If a loss-stopper wins the initial game she plays another one, and thus the initial game is classified as a first game. In contrast, if a loss-stopper loses the initial game she stops playing, and thus the initial game is classified as an only game. Therefore, using the extreme types, a loss-stopper’s winning percentage for first-games must be 100, and by definition the winning percentage for last-games must be 0. Similarly, for win-stoppers, the winning percentage in first-games must be 0, while the winning percentage in last-games must be 100. Combining these two observations leads to the following prediction:

**Prediction 2** The correlation between the winning percentages for first-games and last-games is negative.

Figure 3b presents a scatter plot of winning percentages for first-games and last-game. A strong and significant negative relationship implies that individuals who are more likely to stop playing on a win (loss) have a lower winning (losing) percentage in first-games, as stated in Prediction 2.

For comparison, Figure 3c presents a scatter plot of the winning percentages in last-games and the winning percentages in all games. Notably, there is lower dispersion in the winning percentages for all games than in the winning percentages for first, middle, last, and only-games. In addition, in Figure 3c, the types are not clustered above or below the average winning percentage of around 50%, which indicates that none of the types are better chess players than the others.

Based on the diagram in Figure 2, we can formulate four more predictions about the relationships between the winning percentages in first, middle, last, and only-games. These predictions accurately match the data; see Figure 3d, which presents the correlation matrix with \( p \)-values in parentheses.

### 3 The Model

In this section, we first describe the model with three distinct types of players. We then outline the identification of players’ types, winning percentages, matching probabilities, and utilities from playing a game.
3.1 Description

Here we lay out a chess player’s dynamic choice problem. Let $y_t$ denote the player’s rating at time $t$, which is observable to the player, the player’s opponents, and the econometrician. We assume that $y_t$ lives in finite space $Y$. A player can be one of the following three types: win-stopper ($\theta_W$), loss-stopper ($\theta_L$), or neutral ($\theta_N$). Let $\Theta = \{\theta_W, \theta_L, \theta_N\}$ be the set of all types and let $\theta$ be an element of this set. The player’s type is fixed over time. A player’s type profile at time $t$, denoted as $(y_t, \theta)$, consists of the player’s time-variable characteristics, $y_t$, and a fixed unobservable type, $\theta$. We use variables without time subscripts to denote current states and use “prime” superscripts to denote the next period’s state.

Each period, a player faces the following decision: given the history of the previous game, the player needs to decide whether to play an additional game or to go offline and take the outside option. Before making this decision, the player’s expected utility from playing an additional game is:

$$U(\theta, y, \chi) = u(y) + (1 - \chi)l_\theta$$

(1)

where $y$ is the player’s current rating, $\theta$ is the player’s type, and $\chi$ is the outcome of the last game. If a player won the last game ($\chi = 1$), the utility from playing another game is $u(y)$. This term represents how much the player enjoys playing chess independently of her type. If the player lost the last game, then her utility from playing another game depends on her type.

**Definition 2** A player is a behavioral type if $l_\theta > 0$ or $l_\theta < 0$. She is

- a win-stopper if $l_\theta > 0$;
- a loss-stopper if $l_\theta < 0$.

A player is a neutral type if $l_\theta = 0$.

There is an outside option, $c$, which is independently drawn from a distribution with density $f(c)$ in every period. If a player ends a session, she takes the outside option $c$. If the player does not end the session, her utility is $U(\theta, y, \chi)$ from playing a new game and she moves to the next period, at which point she faces the same decision based on game history updated with the result of the last game she played ($\chi'$). In each period (after a game is over), a player’s decision problem leads to the following Bellman equation,
\[ V(\theta, y, \chi, c) = \max \{ c, u(y) + (1 - \chi)l_\theta + \delta \sum_{y', \chi' \in Y \times \{0, 1\}} p(y', \chi'|y) V(\theta, y', \chi') \}, \tag{2} \]

where \( \delta \) is the discount factor and \( p(y', \chi'|y) \) is the joint probability of the player receiving (transitioning to) the rating \( y' \) and the outcome of the next game being \( \chi' \), conditional on the player’s current rating being \( y \). We have:

\[ p(y', \chi'|y) = \sum_{y-i \in Y} p(y, y-i) p(y'|y, y-i, \chi') p(\chi'|y, y-i) \tag{3} \]

where \( p(y, y-i) \) is the probability that, conditional on a player having rating \( y \), she is matched with a player with rating \( y-i \); \( p(y'|y, y-i, \chi') \) is the probability of receiving (transitioning to) rating \( y' \) given that the player’s current rating is \( y \), in the next game she is matched with a player with rating \( y-i \), and the outcome of the next game is \( \chi' \). Note that we can directly recover \( p(y, y-i) \), \( p(y'|y, y-i, \chi') \), and \( p(\chi'|y, y-i) \) from the data. In our counterfactual analysis, a player-to-player matching mechanism, \( p(y, y-i) \), is a lever market designers can use to influence a player’s decision to start a new game.

### 3.2 Identification

The identification and estimation of the theoretical model follow the tradition of Hotz and Miller (1993). We show that we can forgo numerical dynamic programming to compute the value functions for every parameter vector and we propose an estimation procedure that is simple to implement and computationally efficient. More details and most of the proofs are relegated to Appendix A.

We start with the identification of behavioral types. We first show that an optimal stopping rule is a threshold rule, and then show that these thresholds are i) higher after loss than win for win-stoppers, ii) lower after loss than win for loss-stoppers, iii) equal after loss and win for neutral types. This result suggests that for a given player, if we compare the probability of stopping a session after a win to the probability of stopping a session after a loss, we can determine the behavioral type of that player.

**Claim 1** The optimal stopping rule is a threshold rule in \( c \).

**Proof.** Note that in equation (2), continuation values do not depend on the current realization of \( c \). Hence, fixing the continuation values and current period utility from playing
another game, the second term under the max operator is lower than the outside option, \( c \), for sufficiently high \( c \). Thus, we have a threshold, \( \bar{c}(\theta, y, \chi) \), above which the player stops playing and takes the outside option.

Therefore, \( \bar{c}(\theta, y, \chi) \) is a threshold such that a player with type profile \((\theta, y)\) who has an outcome \( \chi \) in the last game ends a session if and only if the realized \( c \) is at least as large as \( \bar{c}(\theta, y, \chi) \). Recalling equation (2), we have,

\[
\bar{c}(\theta, y, \chi) = u(y) + (1 - \chi)l_\theta + \delta \sum_{y', \chi' \in Y \times \{0, 1\}} p(y', \chi'|y)V(\theta, y', \chi') \quad (4)
\]

The following proposition leads to the identification of behavioral types.

**Proposition 1**

i) \( \bar{c}(\theta_W, y, 0) > \bar{c}(\theta_W, y, 1) \);

ii) \( \bar{c}(\theta_L, y, 0) < \bar{c}(\theta_L, y, 1) \);

iii) \( \bar{c}(\theta_N, y, 0) = \bar{c}(\theta_N, y, 1) \).

**Proof.** The proof follows from equation (4) and definition (2).

**Proposition 1** implies that win-stoppers’ (loss-stoppers’) probability of playing another game is higher if they lost (won) the previous game than if they won (lost). For neutral types, the probability of playing another game is the same no matter the outcome of the last game. Following **Proposition 1**, we can identify a player’s type from the data based on her stopping probabilities after wins and losses. Probabilities of winning and matching probabilities are non-parametrically identified using the data.

To identify the remaining parameters of the model, we assume that the outside option has a parametric distribution and use an exponential distribution for estimation. Under this assumption, \( l_\theta \) and the value of continuing a session are identified based on stopping probabilities after wins and losses. The value function of a player depends on the values from continuing a session and from the outside option distribution parameter. Provided that we identify the value from continuing a session and we normalize the distribution parameter, we identify the value functions. Finally, we show that \( \delta \) and the utilities from playing a game are identified using the player’s value from continuing a session and her value function. See Appendix A for more details on the identification as well as the relevant claims and corresponding proofs.
4 Estimation and Counterfactual Analysis

We begin this section by describing the rating system used on the platform. Next, we present the structural estimation results. Finally, we present the results of a counterfactual analysis, which highlight how a market designer can change a player’s likelihood of staying on the platform.

**Rating system on chess.com** When a player signs up for chess.com, she receives an initial rating. During the data collection period, the default initial rating was 1200 (in January 2021, the default initial rating was changed to 800). A user’s rating changes after every rated game based on the outcome of the game and the opponent’s rating. Intuitively, the rating goes up after a win and down after a loss. Thus, a user’s current rating reflects her current chess expertise: the higher the rating, the better the player. We recover the rating updating rule from the data.

In the following two subsections, we discuss the results of structural estimation analysis and counterfactual analysis using a sub-sample of the data.\(^\text{14}\) We impose two conditions. First, we include only blitz games to make the time spent per game more homogeneous.\(^\text{15}\) Second, we include only the games in which the user’s pre-game rating is between 1000 and 1600.\(^\text{16}\) We impose the second restriction to ensure that users in the sample are homogeneous with respect to their ratings for two reasons. First, we do not want to consider matching of two users with large rating differences. Second, to avoid missing values in the rating transition matrix.\(^\text{17}\) After restricting the data, we end up with 9,192,795 observations for 10,395 unique users.

The next step in the structural estimation analysis is to divide the rating range into grids. For this step, we use the average rating change. In the main sample, the average rating increase after a win is 8.02 points and the average decrease after a loss is 7.97 points. Accordingly, we divide the rating space \([1000, 1600]\) into 8-point intervals, which create a

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\(^{14}\) The structural estimation results using the entire data set are presented in Appendix B.3.

\(^{15}\) On chess.com, blitz is a type of chess game in which each user has a restricted amount of time (3 to 10 minutes) for the entire game.

\(^{16}\) We select this range based on the average and standard deviation of blitz rating in the data. The average blitz rating is 1303 and the standard deviation is 324; we round these to 1300 and 300, respectively, thus obtaining a rating range \([1300-300, 1300+300]\).

\(^{17}\) Consider a user with a rating 700. In our data it is unlikely that this user has ever played against an opponent with rating of 2000. When we calculate what will be a new rating of a user with rating 700 after playing against a user with rating of 2000, we need to consider all such games in the data. Since there may not be a single game with such drastically different ratings we get missing values. To minimize error from estimation we restrict attention to homogeneous users for which we have sufficient amount of data.
4.1 Structural Estimates

Proposition 1 implies that for neutral types, stopping probability is the same after a win and a loss. We do not have an infinite number of observations for each user and therefore we set a tolerance level when we define behavioral types based on the identification provided by Proposition 1. We re-define neutral types as users whose stopping probability after wins and losses is \( \kappa \)-close, that is \(|Pr(Stop|Win) - Pr(Stop|Loss)| \leq \kappa \).\(^18\) Similarly, we modify the win-stopper and loss-stopper definitions such that a user is a win-stopper (loss-stopper) if \( Pr(Stop|Win) - Pr(Stop|Loss) > \kappa \) (\( Pr(Stop|Win) - Pr(Stop|Loss) < -\kappa \)). In this section we use \( \kappa = 0.07 \). Appendix B.4 presents the estimation results using \( \kappa = 0.05 \) and \( \kappa = 0.09 \). Appendix B.2 presents the results of the model type decomposition as \( \kappa \) is varied between 0 and 0.2.

Our estimation strategy parallels the identification proof. First, we estimate \( H(y, \theta) \) and \( l_{\theta} \) using an empirical counterpart of equation (7) in the appendix. Second, we recover \( V(y, \theta, \chi) \) using equation (10). Finally, we use empirical counterparts of the equations in Claim 4 to estimate \( \delta \) and \( u(y) \). We focus on parameters: \( l_{\theta} \) for \( \theta \in \{\theta_W, \theta_L, \theta_N\} \). To check the stability of the results, we bootstrap the data 300 times; the resulting estimates are presented in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_{\theta_W} )</td>
<td>0.678</td>
<td>0.005</td>
</tr>
<tr>
<td>( l_{\theta_N} )</td>
<td>-0.014</td>
<td>0.003</td>
</tr>
<tr>
<td>( l_{\theta_L} )</td>
<td>-0.610</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 2: Bootstrapped values for \( l_{\theta} \)

Table 2 shows that for win-stoppers, the utility from playing another game is 0.678 higher after a loss than after a win. The effect is opposite for loss-stoppers: the utility from playing another game is 0.610 lower after a loss than after a win. Intuitively, for neutral types, the result of the last game has no sizable effect on utility.

4.2 Counterfactual Analysis

Can a market designer leverage information on behavioral types to increase a user’s expected number of games played on the platform? To answer this question, we need to

\(^{18}\) We do not assume \( \kappa \) is the same as \( \tau \), which is the tolerance level we introduced in Definition 1.
understand which factors the market designer can control. The platform can alter the user-to-user matching algorithm. Therefore, we need to know the current matching algorithm, which we recovered directly from the data.

The platform has a higher probability of matching users with closer ratings. In other words, the platform decides which two users are matched based only on users’ ratings. In the counterfactual exercise, we allow the platform to choose from matching mechanisms that are contingent on not only users’ ratings but also their behavioral type.

This section aims to identify a matching algorithm that improves the likelihood of users staying on the platform. Ideally, we would identify the optimal matching algorithm among all those that account for the users’ rating and type. However, due to the problem’s high dimensionality (there are $3n(n - 1)$ variables to optimize, where $n$ is the number of rating grids), our current computational capabilities do not allow us to find the best algorithm. Instead, we answer the following question: how does the expected session length change when we adjust the probability of winning by changing the matching algorithm on the platform? Using structural estimates from Table 2, we conducted several counterfactual analyses to answer this question.

Figure 4 presents the percentage change in average session length (x-axis) by changing the winning percentage (y-axis), which in turn is a result of changing the matching algorithm. The winning percentage for neutral types should be around 50% (the red line). From the definition of behavioral types, a decrease in the winning percentage for win-stoppers and an increase in the winning percentage for loss-stoppers should increase average session length. Figure 4 confirms this intuitive assumption. Changing the matching algorithm by matching win-stoppers (triangles in Figure 4) with, on average, increasingly higher rated opponents decreases their winning percentage but increases average session length. For example, using a matching process that decreases the winning percentage from 50% to 45% increases the average session length by 3.75%. Using a matching algorithm that drops the winning percentage to 40% increases the average session length by around 6%. Similarly, for loss-stoppers, using a matching algorithm that increases the chances of winning from 50% to 60% increases the average session length by 1%. Using a matching process that increases winning percentage from 50% to 65% increases the average session length by

---

19 We did optimize over a smaller class of matching algorithms with limited 3-rating grid. We find that the platform can increase the session length by around 62%. However, the identified matching algorithm matches users with very low ratings to users with very high ratings. Such a drastic change in the matching algorithm could lead users to learn about the matching process or become uninterested in a platform that frequently matches them with a much stronger or weaker opponent.
more than 7.5%.

![Figure 4: Winning percentage and percentage change in session length](image)

Let us put the numbers from Figure 4 in context. In the sample of homogeneous blitz games, an average user played 274 sessions per year. An average session lasted about 3.29 games. The average blitz game in the sample lasted 7 minutes and 29 seconds. Thus, over one year, a 5% increase in session length results in an average user playing 45 more games or spending 6 hours and 37 minutes longer on the platform.

Importantly, changing the winning percentage does not change the distribution of win-stoppers’ and loss-stoppers’ ratings. For example, it might seem that since win-stoppers are winning less often than loss-stoppers, the latter group will accumulate higher ratings. However, this is not the case in the counterfactual analyses because of the inclusion of equation 2. The transition probability $p(y', \chi'|y)$ is defined as $\sum_{y_{-i} \in Y} p(y, y_{-i}) p(y'|y, y_{-i}, \chi') (p(\chi'|y, y_{-i})$, which shows that transition probability depends on not only a user’s current rating and the outcome of the game but also the opponent’s rating. Winning against a more highly rated user increases one’s rating more than winning against a weaker user, and similarly, losing to a weaker user decreases one’s rating more than losing to a stronger user. Therefore, while win-stoppers win less often than loss-stoppers in the counterfactual exercises, they are playing against, on average, stronger opponents than loss-stoppers, which balances the average accumulated rating.

---

20. The average number of sessions and average session length differ from those in Table 1 because the structural estimation analysis includes only blitz games played in 2017, while the table includes all games played in 2017 and 2018.
5 Time Stability and Modeling Choices

In this section we consider tests of the robustness of the main environment to a variety of choices and alternative specifications. We start by testing how a user’s type changes over time; we then test model-related assumptions.

5.1 Time Stability of Behavioral Types

For most of the analyses, we utilize data from 2017. However, we also have data for the same users for 2018, which we use to check the time stability of behavioral types. In Section 2, we classify subjects into types based on the 2017 data. Using the 2018 data to classify the same sample of users, we find a 78.1% match. Thus, 78.1% of users are identified as the same behavioral type in 2017 and 2018. We conducted additional analyses to examine transitions from year to year. Figure 5a presents the transition matrix between types from 2017 to 2018. Neutral types are most likely to experience a shift in classification. This result is not surprising since the definition of types is based on a threshold level with a tolerance of 7%, and movements happen near the threshold.

While collecting the data, we did not place any restrictions on users’ history. Some users played numerous games in 2017 and only a few in 2018, implying that the user’s behavioral classification in 2017 is more accurate than the one in 2018 (due to the number of observations for this user). In addition, some users started playing late in 2017 (and therefore played few games) but played many games in 2018. Due to this heterogeneity, we observe some movement in the transition matrix. However, notice that transitions between behavioral types happen at most for 2% of users.

5.2 Consistency of Behavioral Types

In this paper we propose two distinct definitions of behavioral types (Definitions 1 and Definition 2). The two classifications are intuitively related, but do not necessarily overlap. That is, given some data, a user can be classified as a win-stopper according to Definition 1, but be identified as a loss-stopper by the model (Definition 2 based on Proposition 1). For example, consider a user whose complete playing history consists of the following set of three sessions:

\{WWWW, WLW, WLL\}.

According to Definition 1, the user is classified as a win-stopper because she won last-
games more often than middle-games. For this user, the stopping probability after a loss is $\Pr(\text{Stop}|\text{Loss}) = 1/3$ and the stopping probability after a win is $\Pr(\text{Stop}|\text{Win}) = 2/7$. Given that the probability of stopping is higher after a loss than after a win, $\Pr(\text{Stop}|\text{Loss}) > \Pr(\text{Stop}|\text{Win})$, our model would identify the user as a loss-stopper. The fact that one definition does not necessarily imply the other strengthens any relationship we find between the two classifications, thus highlighting the consistency between our intuition and the proposed theory. Let us compare the two classifications.

For 84.6% of the users the two classifications match. This result provides strong evidence that the model captures users’ behavior and that the game outcome affects the utility of the next game. Figure 5b presents a transition matrix for model types and behavioral types. We observe a large mass on the diagonal, indicating that the two classifications are fairly consistent. For example, 91% of win-stoppers identified by the model were also identified as win-stoppers under Definition 1. However, there are some mismatches; for example, some neutral types by Definition 1 are classified as behavioral types by the model and vice versa. Notably, cases in which a win-stopper (loss-stopper) under Definition 1 is identified as a loss-stopper (win-stopper) by the model are rare, occurring only about 1% of the time.

![Figure 5: Time stability and model consistency](image)

### 5.3 Validity of Modeling Choices

Until this point, we have focused on the effect of the last game’s outcome on the decision to start a new game. In this section, we explore other factors that may affect stopping decisions. We estimate a Cox proportional hazards (CPH) model to evaluate the impact of several factors on survival, which in this setting means not ending a session. Survival anal-
ysis allows us to examine how the specified factors, which are called covariates, influence the session stopping rate, which is referred to as the hazard rate.

We use a Cox proportional hazards model with time-dependent covariates. The general description of the model is as follows:

$$h_j(t, x_j(t)) = h_0(t) \exp \{x_j(t)' \beta\}.$$  

(5)

The left-hand side (LHS) of equation (5) represents the risk that game $j$ with characteristics $x_j(t)$ is the last game of the session (i.e., the session stops after that game). The right-hand side (RHS) of the equation consists of two components: baseline risk and relative risk. The baseline risk, $h_0(t)$, represents the risk that a game will be the last game in a session when all covariates equal zero, $x_j(t) = 0$. The relative risk, $\exp \{x_j(t)' \beta\}$, is the proportionate increase or reduction in risk associated with the set of characteristics $x_j(t)$.

Table 3 presents the CPH results using the pooled data to examine the effect of the last game on the decision to stop a session.

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Coef</th>
<th>exp(Coef)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome</td>
<td>-0.28</td>
<td>0.76</td>
<td>&lt; 0.005</td>
</tr>
</tbody>
</table>

Table 3: CPH without type heterogeneity

The variable Outcome is 1 if a user won the last game, and 0 otherwise. It is easier to interpret the results using the information in the third column: $\exp$(Coef). Specifically, $\exp$(Coef) = 1 implies that the last game’s outcome does not affect the decision to stop the session. In our estimation, the value of $\exp$(Coef) is 0.76, which indicates that a user is 24% $((1 - \exp$(Coef))·100%) less likely to stop playing after a win than after a loss. Recall that this behavior is observed among loss-stoppers but not among win-stoppers (who are less likely to stop playing after a loss). Thus, the result in Table 3 obscures an important heterogeneity that was revealed by the analyses in the previous sections of the paper. The negative relationship we observe occurs because more users are loss-stoppers than win-stoppers.

We next introduce behavioral type heterogeneity among users based on the model estimation in Section 3. To ease the interpretation of results, we assume there are no neutral types and we have only two types of users: win-stoppers and loss-stoppers. We include the user’s type, the interaction of types with the last game outcome as covariates and re-
estimate the CPH model. Table 4 presents the results.

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Coef</th>
<th>exp(Coef)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome</td>
<td>-0.58</td>
<td>0.56</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type</td>
<td>-0.63</td>
<td>0.53</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type × Outcome</td>
<td>1.07</td>
<td>2.91</td>
<td>&lt; 0.005</td>
</tr>
</tbody>
</table>

Table 4: CPH with type heterogeneity

The variable *Type* is 1 for win-stoppers and 0 for loss-stoppers. Therefore, the baseline in the estimation is a loss-stopper who lost the last game. Table 4 shows that for a win-stopper, the hazard rate is higher after a win than after a loss.\(^{21}\) In other words, for a win-stopper, the chance of ending a session is lower after a loss than after a win. This is the expected result. We observe the reverse relationship for loss-stoppers: a win in the last game decreases the hazard rate by 44% more than a loss (baseline).

5.3.1 History dependence beyond the last game

The focus of this paper has been the effect of win/loss history in the last game. It is possible that the outcome in the game before the last game also affects stopping decisions. Some users may be more likely to stop after two wins in a row (rather than just one) while others may be more likely to stop after two losses in a row. In this section, we examine an effect of the outcomes of the last two games. We estimate the CPH model as before, but now add a lagged outcome (Lag 1 Outcome) as well as the interaction of this variable with user’s type and the outcome of the last game. Table 5 presents the estimation results.

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Coef</th>
<th>exp(Coef)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome</td>
<td>-0.62</td>
<td>0.54</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type</td>
<td>-0.65</td>
<td>0.52</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type × Outcome</td>
<td>1.13</td>
<td>3.08</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Lag 1 Outcome</td>
<td>0.04</td>
<td>1.04</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type × Lag 1 Outcome</td>
<td>-0.04</td>
<td>0.96</td>
<td>&lt; 0.005</td>
</tr>
</tbody>
</table>

Table 5: CPH with type heterogeneity and Lag 1 outcome

Adding the outcome of the game prior to the last game has little effect on the estimates. The outcome of the last game has effects similar to those observed in Table 4. The effects

\(^{21}\) For a win-stopper, the hazard rate is lower by 0.14 (-0.58-0.63+1.07=-0.14) after a win (compared to baseline) and lower by 0.63 after a loss (compared to baseline). Therefore, a loss decreases the hazard rate much more than a win.
of lagged values are statistically significant but considerably smaller in magnitude than the effects of the last game’s outcome. Notably, the sign of the lagged variable’s effect is predicted by the arguments outlined in Section 2.3.2.

5.3.2 Initial rating as a reference point

One factor that may affect a user’s stopping decision is their initial rating in a session. If a user’s stopping rule is to end a session once her current rating surpasses her initial rating for the session, then the rating difference between the first and last-games of a session should be significant. To test this hypothesis, we include rating change since the start of the session in the CPH regression and re-estimate the model.

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Coef</th>
<th>exp(Coef)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome</td>
<td>-0.62</td>
<td>0.54</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type</td>
<td>-0.65</td>
<td>0.52</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type × Outcome</td>
<td>1.11</td>
<td>3.03</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Rating Change</td>
<td>0.05</td>
<td>1.05</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type × Rating Change</td>
<td>-0.05</td>
<td>0.95</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Outcome × Rating Change</td>
<td>0.00</td>
<td>1.00</td>
<td>&lt; 0.36</td>
</tr>
</tbody>
</table>

Table 6: CPH with type heterogeneity and rating change since start of session

The results in Table 6 show that the change in the rating since the beginning of the session affects stopping behavior; however, rating change has a smaller effect than the last game’s outcome. To make this point visually, we run the CPH regression separately for win-stoppers and loss-stoppers; we present the results in Figure 6. The effects of rating change and the interaction between rating change and Outcome are around 0 for both Win- and loss-stoppers, in other words, after controlling for the last game’s outcome, rating change does not have a sizable effect on stopping decisions. For example, Figure 6 shows that the coefficient of Outcome for Win-Stopper is 0.51 (p-value < 0.005), with \( \exp(\text{Outcome}) = 1.67 \), meaning that a win-stopper is 67% more likely to stop after a win than after a loss, while all other coefficients imply no more than 4% change in stopping decision.

5.3.3 Playing against a stronger opponent

Another variable that could affect stopping decisions is the opponent’s rating. Winning against a stronger opponent may be more enjoyable than winning against a weaker opponent. Similarly, losing to a weaker opponent may be more painful than losing to a stronger
one. To test if and by how much winning or losing against stronger or weaker opponents affects stopping decisions, we re-estimate the CPH model with a new covariate: the difference in ratings between the two users.

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Coef</th>
<th>exp(Coef)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome</td>
<td>-0.58</td>
<td>0.56</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type</td>
<td>-0.64</td>
<td>0.53</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type × Outcome</td>
<td>1.08</td>
<td>2.95</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Rating Difference</td>
<td>0.01</td>
<td>1.01</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Type × Rating Difference</td>
<td>-0.01</td>
<td>0.99</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>Outcome × Rating Difference</td>
<td>-0.04</td>
<td>0.96</td>
<td>&lt; 0.005</td>
</tr>
</tbody>
</table>

Table 7: CPH with type heterogeneity and rating difference at start of game

Table 7 shows that users act differently after winning and losing against stronger and weaker opponents; however, this effect is of a much smaller scale than the effect of the last game’s result. Again, Figure 6 provides a visual illustration this result. Both the rating difference and the interaction between rating difference and the last game’s result are statistically significant. Nonetheless, the magnitude of the effect of the last game’s result is much stronger for both win-stoppers and loss stoppers.

Figure 6: CPH coefficients for Win-Stoppers and Loss-Stoppers
6 Conclusion

This paper investigates stopping behavior on an online chess platform and identifies factors that determine how people make stopping decisions. We use rich data collected from chess.com that provide information about an individual’s behavior over two years. Using these data, we identify two behavioral types: win-stoppers and loss-stoppers. Win-stoppers are more likely to stop playing after a win, while loss-stoppers are more likely to stop playing after a loss. We classify 79% of users as one of these behavioral types using conservative parameter values; one-third of this group are win-stoppers and the rest are loss-stoppers. We explore how market designers can leverage information on users’ types to increase the number of games played.

To quantify the impact of the last game’s outcome on stopping behavior, we develop a dynamic discrete choice model in which the agent may have time non-separable preferences. The model allows for the future game utility to depend on the current game outcome and captures heterogeneity in stopping behavior. The estimates from the structural model are consistent with the above-mentioned reduced-form evidence.

We use the structural model estimates to conduct a counterfactual analysis that evaluates alternative market designs. We show that matching win-stoppers with, on average, more challenging opponents increases the average number of games played. For example, a matching procedure that decreases a win-stopper’s winning percentage from 50% to 45% (40%) increases the average number of games played in a session by 3.75% (6%). Conversely, a matching procedure that increases a loss-stopper’s winning percentage from 50% to 60% (65%) increases the average number of games played during a session by 1% (7.5%). To contextualize these numbers, over one year, a 5% increase in session length results in an average user playing 45 more games, which translates to spending 6 hours and 37 minutes longer on the platform.

Online platforms employ several monetization strategies, including advertising, subscriptions, in-app purchases, sponsorship, and freemium upsell. Time spent on the platform affects a number of these strategies. For example, time spent on the platform has a straightforward impact on advertisement—the longer a user is on a platform, the more ads the platform can show the user. Further, one aspect of retaining users is user experience, which is affected by a platform’s ability to find a match in a timely fashion. Using the analytical results in the paper, a platform can increase market thickness, which will be especially beneficial during times with lower activity.
References


Appendices

A Identification proofs

The following parametric assumption is made on the distribution of outside option, $F(c)$,

**Assumption 1** $F(c)$ is an exponential distribution with parameter $\lambda$.

We now argue that under the Assumption 1 and by normalizing one parameter of our choice in the model, we can identify $\delta$, $l_\theta$ and $u(\cdot)$. Let,

$$H(\theta, y) = u(y) + \delta \sum_{y', \chi' \in Y \times \{0, 1\}} p(y', \chi'|y) V(\theta, y', \chi'). \quad (6)$$

Under the Assumption 1 and from equation 4, the probability of stopping and taking outside option, $h(\theta, y, \chi)$, can be written as,

$$h(\theta, y, \chi) = e^{-\lambda H(\theta, y) + (1-\chi)l_\theta} \quad (7)$$

**Claim 2** $\lambda H(\theta, y)$ and $\lambda l_\theta$ are identified for all $(\theta, y)$.

**Proof.** Let us look at equation (7) evaluated at $\chi = 1$. The LHS, $h(\theta, y, 1)$, can be directly calculated from the data as probability of stopping after a win. In the RHS, the second term in the power, $(1-\chi)l_\theta$, is 0. Therefore, we can recover/identify $\lambda H(\theta, y)$ from (7).

Next, let us look at equation (7) evaluated at $\chi = 0$. The LHS can be calculated from the data as probability of stopping after a loss. In the RHS, $H(\theta, y)$ term was identified in the first part of the proof. Thus, $\lambda l_\theta$ is identified from (7) as well. ■

**Claim 3** $\lambda V(\theta, y, \chi)$ are identified for all $(\theta, y, \chi)$.

**Proof.** With a little abuse of notation let us denote $\bar{c}(\theta, y, \chi)$ by $\bar{c}$. We can rewrite (2) as,

$$V(\theta, y, \chi, c) = 1(c > \bar{c}) * c + 1(c \leq \bar{c}) \left( H(\theta, y) + (1-\chi)l_\theta \right), \quad (8)$$

where $1(\cdot)$ is an indicator function. Taking expectations of both hand sides of (8) with
respect to $c$ gives,

\[
V(\theta, y, \chi) = E(c|c > \bar{c}) + Pr(c \leq \bar{c}) \left( H(\theta, y) + (1 - \chi)l_\theta \right)
\]

\[
= Pr(c > \bar{c}) \left( E(c) + \bar{c} \right) + Pr(c \leq \bar{c}) \left( H(\theta, y) + (1 - \chi)l_\theta \right)
\]

\[
= Pr(c > \bar{c}) \left( \frac{1}{\lambda} + \bar{c} \right) + Pr(c \leq \bar{c}) \left( H(\theta, y) + (1 - \chi)l_\theta \right)
\]

\[
= Pr(c > \bar{c}) \left( \frac{1}{\lambda} + \bar{c} - H(\theta, y) - (1 - \chi)l_\theta \right) + H(\theta, y) + (1 - \chi)l_\theta.
\]

(9)

Multiplying both hand sides by $\lambda$ and substituting $\bar{c}(\theta, y, \chi)$ from expression (4) we get,

\[
\lambda V(\theta, y, \chi) = e^{-\lambda[H(\theta, y)+(1-\chi)l_\theta]} + \lambda H(\theta, y) + (1 - \chi)\lambda l_\theta.
\]

(10)

Claim 2 and expression (10) imply that $\lambda V(\theta, y, \chi)$ are identified for all $(\theta, y, \chi)$. ■

Claim 4 $\delta$ and $\lambda u(y)$ are identified.

Proof. We can consider the difference $\lambda(H(\theta, y) - H(\theta', y))$ for some $\theta \neq \theta'$. This gives us,

\[
\delta = \frac{\lambda(H(\theta, y) - H(\theta', y))}{\lambda(\sum_{y', \chi' \in Y \times \{0,1\}} p(y', \chi'|y)(V(\theta, y', \chi') - V(\theta', y', \chi')))}.
\]

By claims 2 and 3, numerator and denominator are identified in the above equation. Finally, we can identify $\lambda u(y)$ from,

\[
\lambda u(y) = \lambda H(\theta, y) - \lambda \delta \sum_{y', \chi' \in Y \times \{0,1\}} p(y', \chi'|y) V(\theta, y', \chi')
\]

Finally, we can identify all the parameters and value functions by normalizing $\lambda$. This completes the identification of the parameters of the model.

B Robustness

B.1 Changing session definition—varying break time

In the main body of the paper, while defining a session, we set the break time $T$ to 30 minutes. To ensure that the results on the behavioral types are not sensitive to the choice of $T$,
we classify users into types using sessions defined by break times $T \in \{5, 15, 30, 60\}$. We are interested in how the behavioral type classification changes and the transition between the different $T$s. Figure 7 presents transition matrices.

In Figure 7 we see a large mass on the diagonal, which implies that the classifications mostly match. However, there are some mismatches; for example, some neutral types with $T = 5$ are classified as behavioral types with $T = 15$. What is noteworthy in the panel are the transitions between behavioral types: 0% of users are classified as a win-stopper (loss-stopper) by one classification and a loss-stopper (win-stopper) by another classification or vice versa. There are no switches in behavioral types as we vary $T \in \{5, 15, 30, 60\}$.

![Transition Matrices]

Figure 7: Transitions between 5 to 15, 15 to 30, 30 to 60, and 5 to 30

### B.2 Robustness of tolerance thresholds

Unless we stated otherwise, throughout the paper, we set tolerance levels $\tau$ and $\kappa$ to 7%. We vary $\tau$ and $\kappa$ from 0 to .2 and we classify our users into types according to Definition 1 and model identification. Figure 8 presents the users’ population decomposition by types with $\tau \in [0, .2]$ and $\kappa \in [0, .2]$. While there is movement in a predicted direction—higher the threshold, less behavioral types—we see that types are overall robust to changing the
allowed tolerance (no abrupt, unexpected discontinuities).

(a) Behavioral Type Fractions

(b) Model Type Fractions

![Figure 8: Type Decomposition](image)

### B.3 Full data structural estimation results

To show that estimation does not depend on the grid size or data range, we change both and compare the results. We divide the rating range into grids of 20 since the rating has a wide range ([100, 2798]). We have few observations where the rating is below 600 or above 2000. Consequently, we place all the users with a rating below 600 in the first rating grid and those above 2000 in the last rating grid (grid 71). We divide the rest of the rating range into 20 point intervals.

The main parameters that we focus on are $l_{\theta}$ for $\theta \in \{\theta_W, \theta_L, \theta_N\}$. The estimates are presented in Table 8. We bootstrapped 300 times to find standard deviation of the parameters. Table 8 shows that parameter estimates as well as their standard deviations are similar to the ones in Table 2 in Section 4.1. Parameter estimates are stable with respect to rating range and the grid size.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{\theta_W}$</td>
<td>0.665</td>
<td>0.004</td>
</tr>
<tr>
<td>$l_{\theta_L}$</td>
<td>−0.017</td>
<td>0.002</td>
</tr>
<tr>
<td>$l_{\theta_N}$</td>
<td>−0.604</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 8: Bootstrapped values for $l_{\theta}$

33
B.4 Robustness with respect to behavioral type tolerance level

From the definition of behavioral types, it is clear that estimates of $l_{\theta}$ for $\theta \in \{\theta_W, \theta_L, \theta_N\}$ depend on behavioral type tolerance level. In the main text all our results are for $\kappa = 0.07$. In this section we present estimation results for two other values $\kappa = 0.05$ and $\kappa = 0.09$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{\theta_W}$</td>
<td>0.622</td>
<td>0.004</td>
</tr>
<tr>
<td>$l_{\theta_N}$</td>
<td>-0.005</td>
<td>0.003</td>
</tr>
<tr>
<td>$l_{\theta_L}$</td>
<td>-0.58</td>
<td>0.002</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
<td>------</td>
</tr>
<tr>
<td>$l_{\theta_W}$</td>
<td>0.737</td>
<td>0.005</td>
</tr>
<tr>
<td>$l_{\theta_N}$</td>
<td>-0.027</td>
<td>0.003</td>
</tr>
<tr>
<td>$l_{\theta_L}$</td>
<td>-0.642</td>
<td>0.003</td>
</tr>
</tbody>
</table>

(a) $\tau = 0.05$  
(b) $\tau = 0.09$

Table 9: Bootstrapped values for $l_{\theta}$

Table 9 shows that parameter estimates changes in the expected direction. For example, when we relax non-behavioral (neutral type) constraint from $\kappa = 0.07$ to $\kappa = 0.09$, there are less behavioral types. Therefore, the users who are still behavioral types with $\kappa = 0.09$, are the ones who are “more behavioral” then the one with $\kappa = 0.07$. This implies that the effect from the last game result (whether negative or positive) is stronger for those behavioral types. This comparative static is met in our estimates. Win-stoppers’ parameter $l_{\theta_W}$ is lower for $\kappa = 0.05$ and higher for $\kappa = 0.09$ compared to $\kappa = 0.07$. Similarly, for loss-stoppers the absolute value of $l_{\theta_K}$ is lower for $\kappa = 0.05$ and higher for $\kappa = 0.09$ compared to $\kappa = 0.07$.

C Possible crowding-out effects

Our counterfactual analysis reveals that considering users’ behavioral type for the matching algorithm can increase the average number of games played during a session. One might think of several crowding-out effects that an increase of a session length might have. Without a randomized controlled trial (RCT) we can not fully address such concerns; however, we provide evidence that some of these effects are not likely.

C.1 More games do not lead to more time on the platform

The goal of the counterfactual analysis is to increase the number of games during a session, but the market designer’s goal could also be to increase the time spent on the platform. We calculated the correlation for every individual between minutes spend on the platform during a session and the number of games played in the same session. Figure 9 shows that
correlation between these two variables is high. The median correlation between minutes and games during the session is 0.98 across users.

![Figure 9: Correlation between minutes spent for a session and number of games](image)

C.2 Asymmetric matching can decrease playing time

Another issue that one might worry about is that asymmetric matching can cause fast games, in the sense that stronger users can win faster playing against a weaker user. To show that this is not likely to be an issue, we calculate the correlation between rating differences and minutes spent on a game. Rating difference provides a measure of how much better one user is compared to another. Figure 10 shows a correlation between how much better an opponent is and how much time the game lasts is close to zero for most cases.

C.3 One long session can cause the next session to be short

One might worry that if a first session time increases during the day, it can decrease the next session length (if users set out a certain amount of time to spend on the platform every day). We find that the correlation between the number of sessions played during a day, and the average length of a session is 0.0002. We also find that one session length does not have any explanatory power on the length of the next session.

C.4 Users adjust game type based on the time they have played

The last issue that we discuss here is changing the type of the game. A person who started a session with a 5-minute blitz game can play a shorter last game (for example 3-minute
game) because she has only a certain time allocated on the platform. If that is the case, we should see that people change game types during the session. We find that 96% of sessions are homogeneous in the sense of the game type. This homogeneity captures not only a change of game type in the last game but during any other time. This makes our argument even stronger that users do not choose the last game type based on the remaining time allocated for playing chess that day.

**D  Winning percentage and game type**

<table>
<thead>
<tr>
<th>Types</th>
<th>O</th>
<th>F</th>
<th>L</th>
<th>M</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss-Stopper</td>
<td>38.5</td>
<td>56.7</td>
<td>35.9</td>
<td>59.2</td>
<td>50.4</td>
</tr>
<tr>
<td>Neutral</td>
<td>50.6</td>
<td>50.0</td>
<td>50.2</td>
<td>50.6</td>
<td>50.5</td>
</tr>
<tr>
<td>Win-Stopper</td>
<td>64.3</td>
<td>43.7</td>
<td>65.6</td>
<td>43.6</td>
<td>50.7</td>
</tr>
</tbody>
</table>

Table 10: Winning percentage

<table>
<thead>
<tr>
<th>Types</th>
<th>O</th>
<th>F</th>
<th>L</th>
<th>M</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss-Stopper</td>
<td>10.93</td>
<td>8.36</td>
<td>8.85</td>
<td>7.59</td>
<td>3.53</td>
</tr>
<tr>
<td>Neutral</td>
<td>9.53</td>
<td>6.86</td>
<td>5.70</td>
<td>5.15</td>
<td>4.21</td>
</tr>
<tr>
<td>Win-Stopper</td>
<td>13.79</td>
<td>9.05</td>
<td>10.53</td>
<td>7.59</td>
<td>4.15</td>
</tr>
</tbody>
</table>

Table 11: Winning percentage standard deviation
E  Rating and behavioral type

In this section, we examine whether a user’s type is correlated with the type classification. In particular, whether a user with a higher rating is more likely to be one type. To study this question, we restrict our data, focusing on users with an average rating of over 1,800 (619 users). Among these users with the best rating, 81.9% are behavioral types. The ratio of win-stoppers among the behavioral types is 32.7%, similar to the full data ratio of 29.8%. A hypothesis that we may observe more of one of the behavioral types among higher rated users, for example, more of win-stoppers, has little support in our data.

F  World maps

The data includes users from 191 different countries, however, there are certain countries with too few user and we exclude those countries. In particular, we remove the countries with less than 30 users and we get 65 countries with at least 30 users. Note, almost 50% of the users in our data indicate their country to be one of the following 5 countries: USA, India, Russia, Canada, or Norway. Figures 11, 12, 13 include three world heat maps showing number of users, average rating and the fraction of win-stoppers among behavioral types.

Figure 11: Number of users from a country

The fraction of behavioral types varies by country and ranges between 67.3% (Finland) and 92.3% (Vietnam). Furthermore, the ratio of win-stoppers among behavioral types varies considerably by country ([10.8, 44.1]). For example, win-stoppers make up

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23 It is important to emphasize, however, that country variable is self-reported by the users, they can choose any country they wish and they can also change it afterwards.
only 10.8% of all behavioral types in Kazakhstan, while in Japan, they are 44.1% of all behavioral types.